MONASH University

MONASH BUSINESS SCHOOL

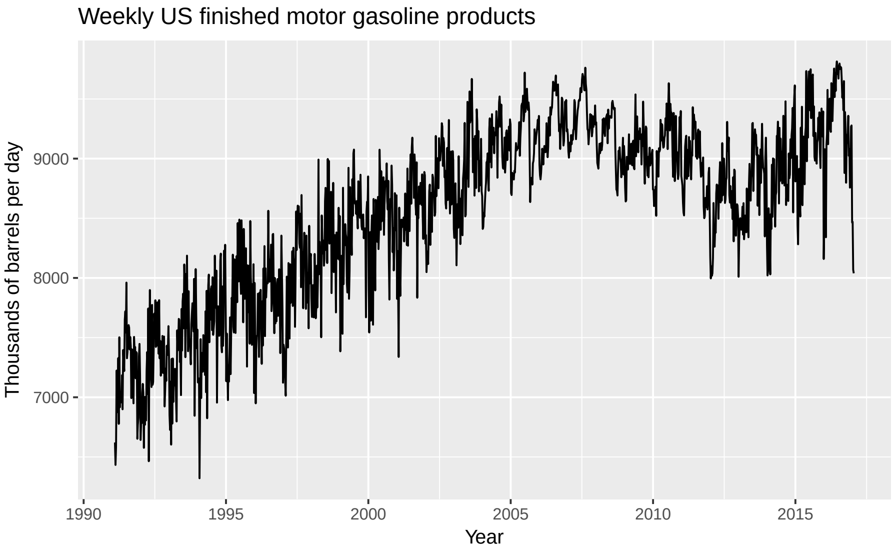
Forecasting: principles and practice

Rob J Hyndman

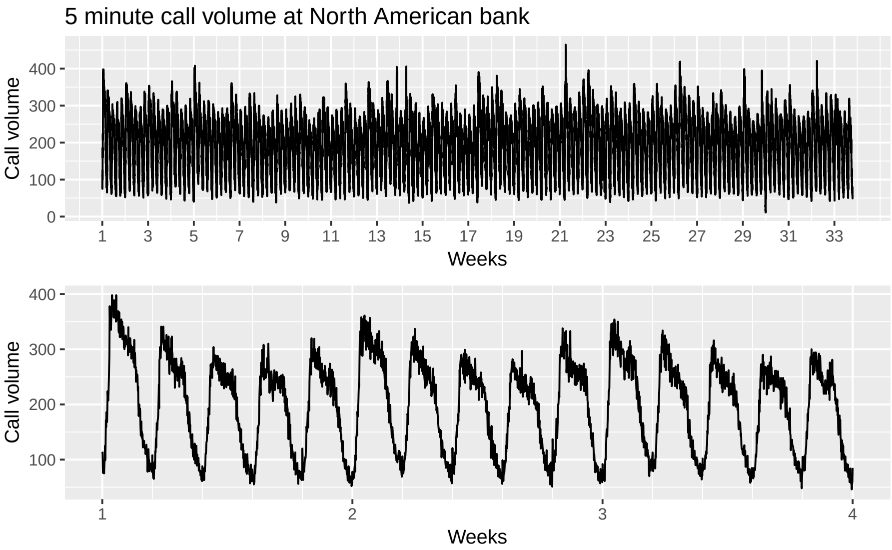
3.4 Advanced methods

Outline

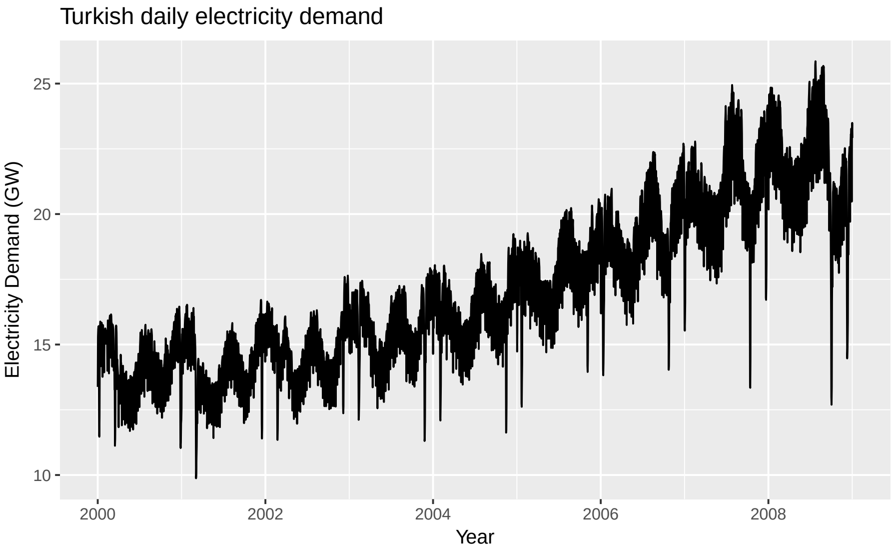
Examples


Weekly US finished motor gasoline products
Thousands of barrels per day
9000
8000
7000
1990
1995
2000
2005
2010
2015
Year

Examples


5 minute call volume at North American bank
Call volume
400
300
200
100
0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
Weeks
Call volume
400
300
200
100
1
2
3
4
Weeks

Examples


Turkish daily electricity demand
Electricity Demand (GW)
25
20
15
10
2000
2002
2004
2006
2008
Year

TBATS model

TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

TBATS model

$$y_t = \text{observation at time } t$$

$$y_t^{(\omega)} = \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

$$\begin{aligned} s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned}$$

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ARMA error

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Fourier-like seasonal terms

TBATS model

y_t = observation at time t

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$y_t^{(\omega)} = \ell_{t-} \ldots d_t$

$\ell_t = \ell_{t-} \ldots$

$b_t = (1 \ldots$

$d_t = \sum_{i=1}^{p} \phi \ldots \sum_{j=1} \ldots$

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TBATS

Trigonometric
Box-Cox
ARMA
Trend
Seasonal

Box-Cox transformation

M seasonal periods

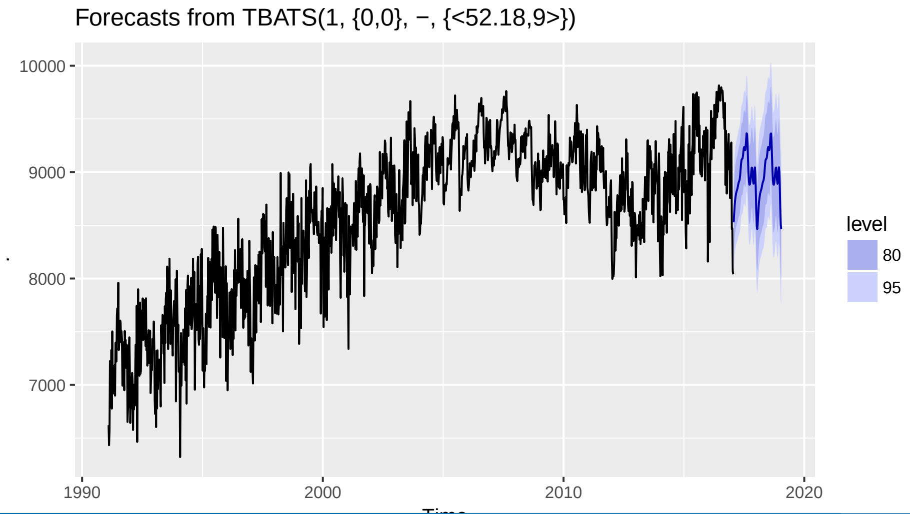
global and local trend

ARMA error

Fourier-like seasonal terms

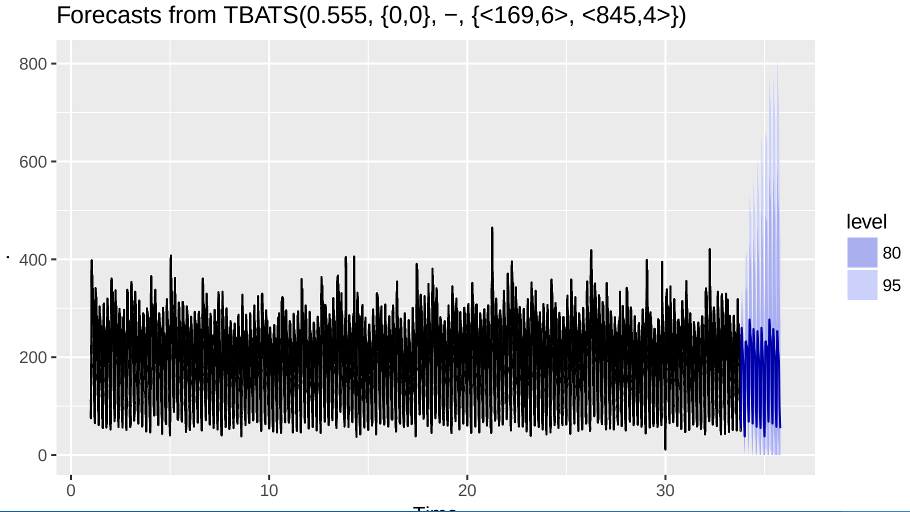
Complex seasonality

```
gasoline %>% tbats %>% forecast %>% autoplot
```



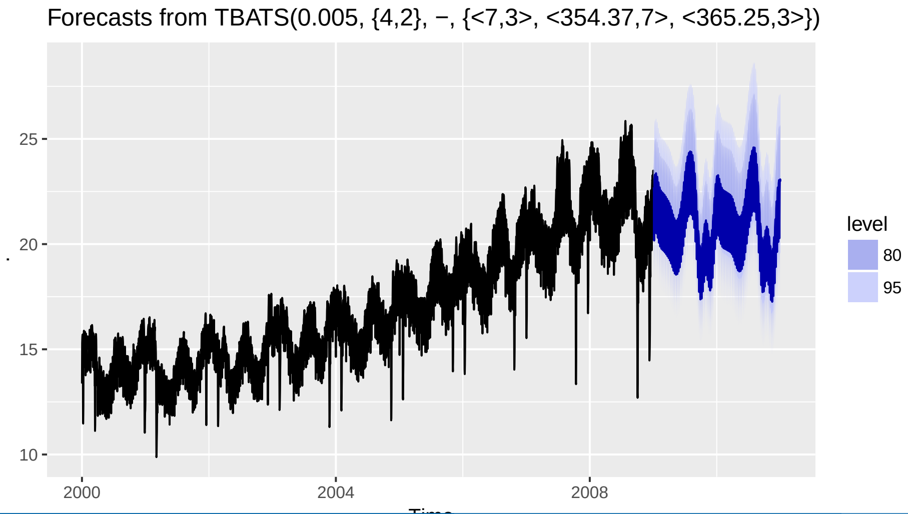
Complex seasonality

```
calls %>% tbats %>% forecast %>% autoplot
```



Complex seasonality

```
telec %>% tbats %>% forecast %>% autoplot
```



TBATS model

TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

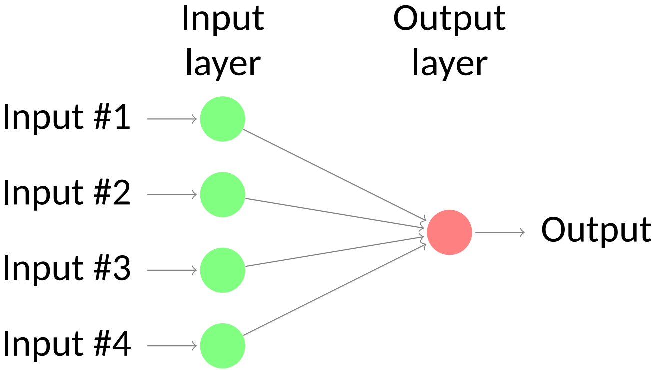
Outline

Lab Session 17

Outline

Neural network models

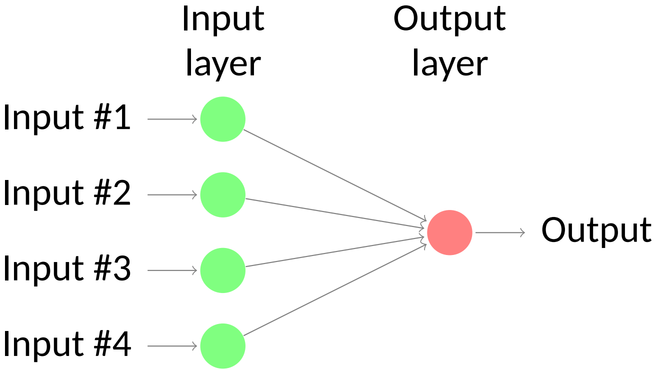
Simplest version: linear regression



- Coefficients attached to predictors are called "weights".
- Forecasts are obtained by a linear combination of inputs.
- Weights selected using a "learning algorithm" that minimises a "cost function".

Neural network models

Simplest version: linear regression



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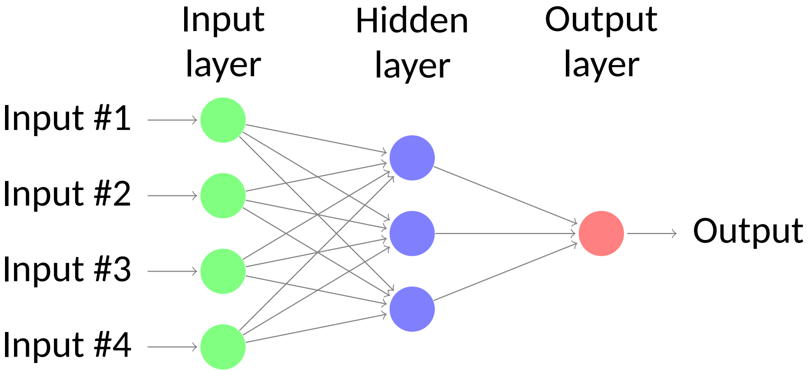
Neural network models

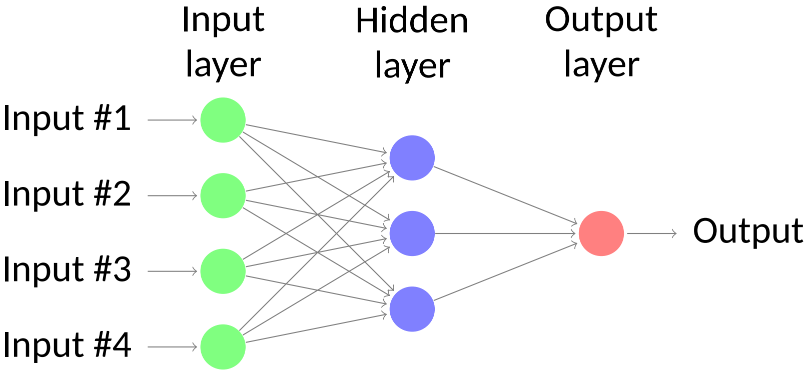
Nonlinear model with one hidden layer



- A **multilayer feed-forward network** where each layer of nodes receives inputs from the previous layers.
- Inputs to each node combined using linear combination.
- Result modified by nonlinear function before being output.

Neural network models

Nonlinear model with one hidden layer



- A **multilayer feed-forward network** where each layer of nodes receives inputs from the previous layers.
- Inputs to each node combined using linear combination.
- Result modified by nonlinear function before being output.

Neural network models

Inputs to hidden neuron j linearly combined:

$$z_j = b_j + \sum_{i=1}^{4} w_{i,j} x_i.$$

Modified using nonlinear function such as a sigmoid:

$$s(z) = \frac{1}{1 + e^{-z}},$$

This tends to reduce the effect of extreme input values, thus making the network somewhat robust to outliers.

Neural network models

- Weights take random values to begin with, which are then updated using the observed data.
- There is an element of randomness in the predictions. So the network is usually trained several times using different random starting points, and the results are averaged.
- Number of hidden layers, and the number of nodes in each hidden layer, must be specified in advance.

NNAR models

- Lagged values of the time series can be used as inputs to a neural network.
- NNAR(p, k): p lagged inputs and k nodes in the single hidden layer.
- NNAR($p, 0$) model is equivalent to an ARIMA($p, 0, 0$) model but without stationarity restrictions.
- Seasonal NNAR(p, P, k): inputs $(y_{t-1}, y_{t-2}, \dots, y_{t-p}, y_{t-m}, y_{t-2m}, y_{t-Pm})$ and k neurons in the hidden layer.
- NNAR($p, P, 0$)$_m$ model is equivalent to an ARIMA($p, 0, 0$)(P,0,0)$_m$ model but without stationarity restrictions.

NNAR models in R

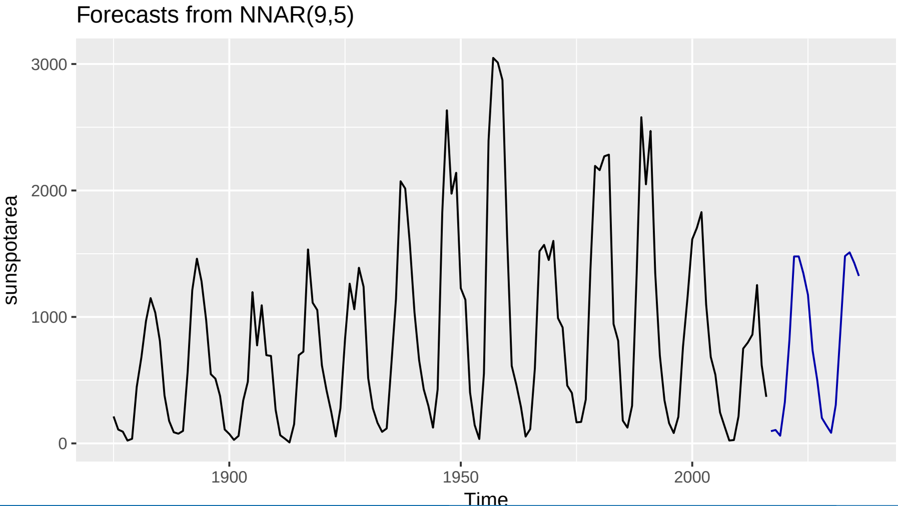
- The `nnetar()` function fits an NNAR$(p, P, k)_m$ model.
- If p and P are not specified, they are automatically selected.
- For non-seasonal time series, default p = optimal number of lags (according to the AIC) for a linear AR(p) model.
- For seasonal time series, defaults are $P = 1$ and p is chosen from the optimal linear model fitted to the seasonally adjusted data.
- Default $k = (p + P + 1)/2$ (rounded to the nearest integer).

Sunspots

- Surface of the sun contains magnetic regions that appear as dark spots.
- These affect the propagation of radio waves and so telecommunication companies like to predict sunspot activity in order to plan for any future difficulties.
- Sunspots follow a cycle of length between 9 and 14 years.

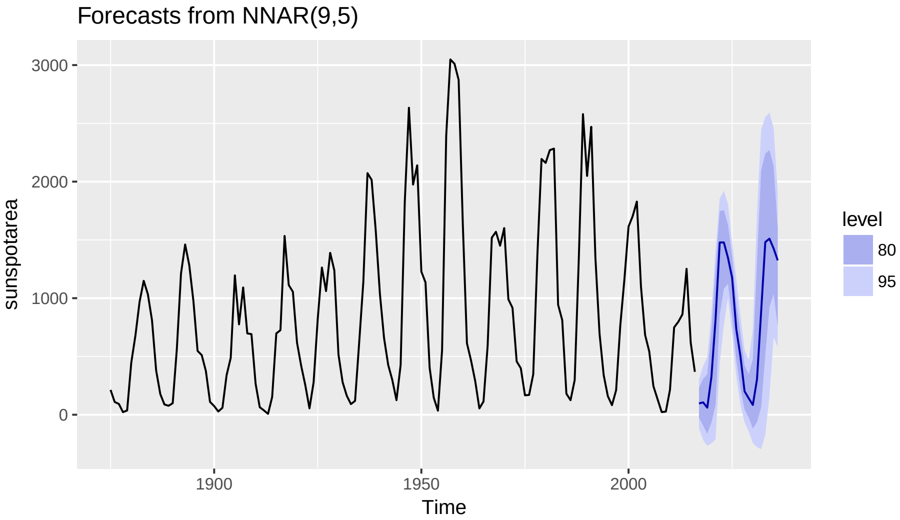
NNAR(9,5) model for sunspots

```
fit <- nnetar(sunspotarea)
fit %>% forecast(h=20) %>% autoplot
```



Prediction intervals by simulation

```r
fit %>% forecast(h=20, PI=TRUE) %>% autoplot
```



Outline

Lab Session 18

Outline

Lab Session 19