MONASH University

MONASH BUSINESS SCHOOL

Forecasting: principles and practice

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1.3 Seasonality and trends

Outline

Time series patterns

Trend pattern exists when there is a long-term increase or decrease in the data.

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

Time series decomposition

$$Y_t = f(S_t, T_t, E_t)$$

where
Y_t = data at period t
S_t = seasonal component at period t
T_t = trend-cycle component at period t
E_t = remainder (or irregular or error) component at period t

Additive decomposition: $Y_t = S_t + T_t + E_t$.

Multiplicative decomposition: $Y_t = S_t \times T_t \times E_t$.

Time series decomposition

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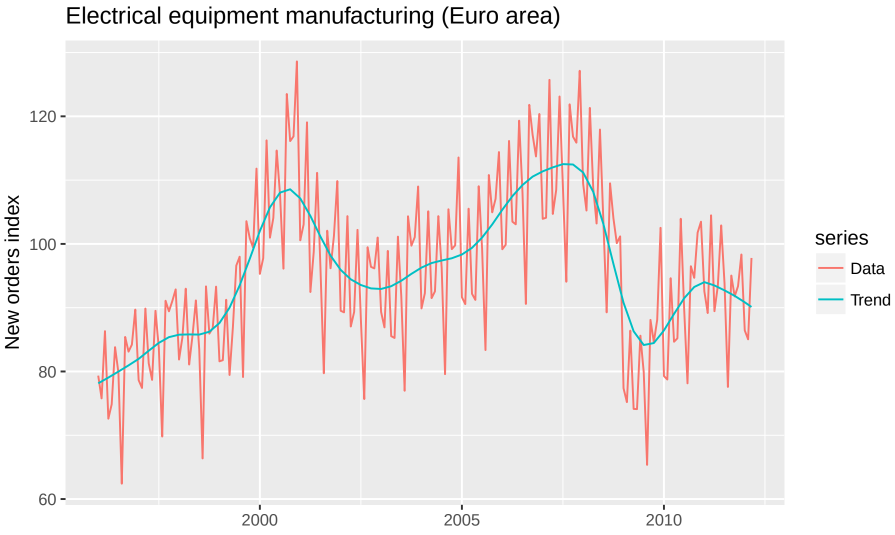
Time series decomposition

- Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.
- If seasonal are proportional to level of series, then multiplicative model appropriate.
- Multiplicative decomposition more prevalent with economic series
- Alternative: use a Box-Cox transformation, and then use additive decomposition.
- Logs turn multiplicative relationship into an additive relationship:

$$Y_t = S_t \times T_t \times E_t \quad \Rightarrow \quad \log Y_t = \log S_t + \log T_t + \log E_t.$$

Euro electrical equipment



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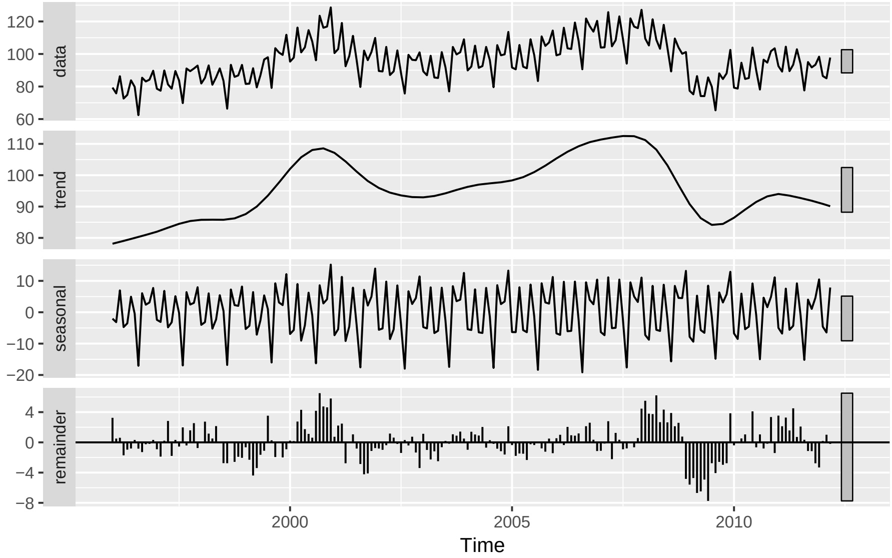

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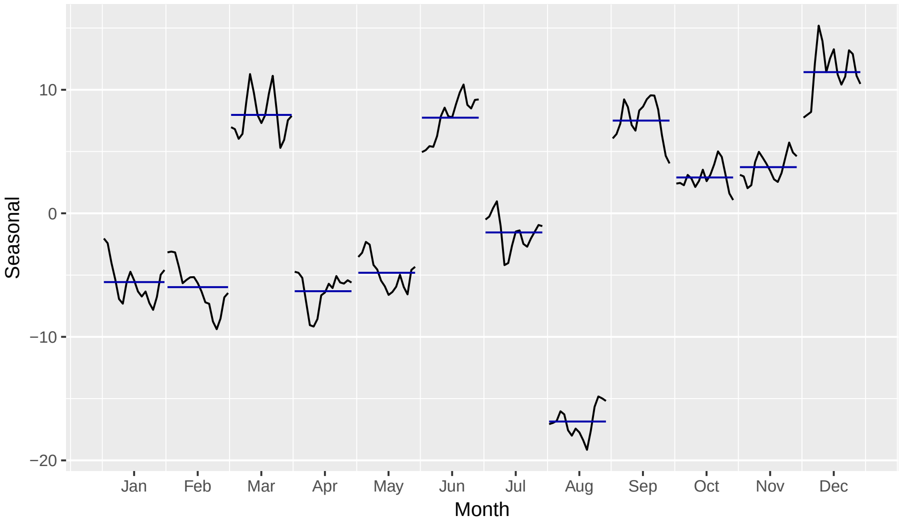

Seasonal adjustment

- Useful by-product of decomposition: an easy way to calculate seasonally adjusted data.
- Additive decomposition: seasonally adjusted data given by
$$Y_t - S_t = T_t + E_t$$
- Multiplicative decomposition: seasonally adjusted data given by
$$Y_t/S_t = T_t \times E_t$$

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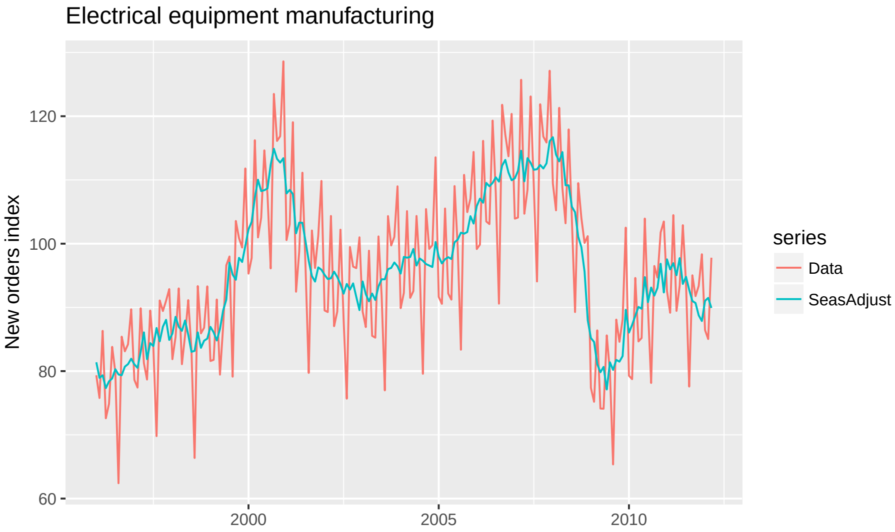

Seasonal adjustment

- We use estimates of *S* based on past values to seasonally adjust a current value.
- Seasonally adjusted series reflect **remainders** as well as **trend**. Therefore they are not “smooth” and “downturns” or “upturns” can be misleading.
- It is better to use the trend-cycle component to look for turning points.

History of time series decomposition

- Classical method originated in 1920s.
- Census II method introduced in 1957. Basis for modern X-12-ARIMA method.
- STL method introduced in 1983
- TRAMO/SEATS introduced in 1990s.

Outline

STL decomposition

- STL: “Seasonal and Trend decomposition using Loess”,
- Very versatile and robust.
- Unlike X-12-ARIMA, STL will handle any type of seasonality.
- Seasonal component allowed to change over time, and rate of change controlled by user.
- Smoothness of trend-cycle also controlled by user.
- Robust to outliers
- Not trading day or calendar adjustments.
- Only additive.

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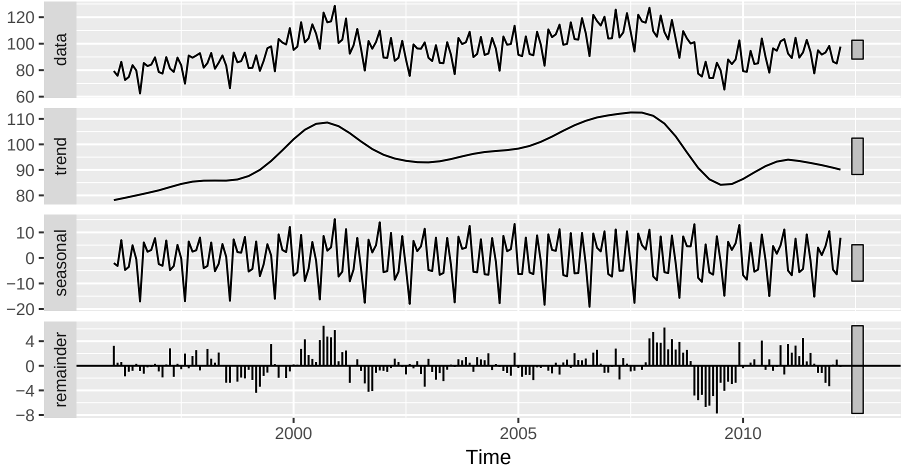
```
elecequip %>% stl(s.window=5) %>%
  autoplot
```



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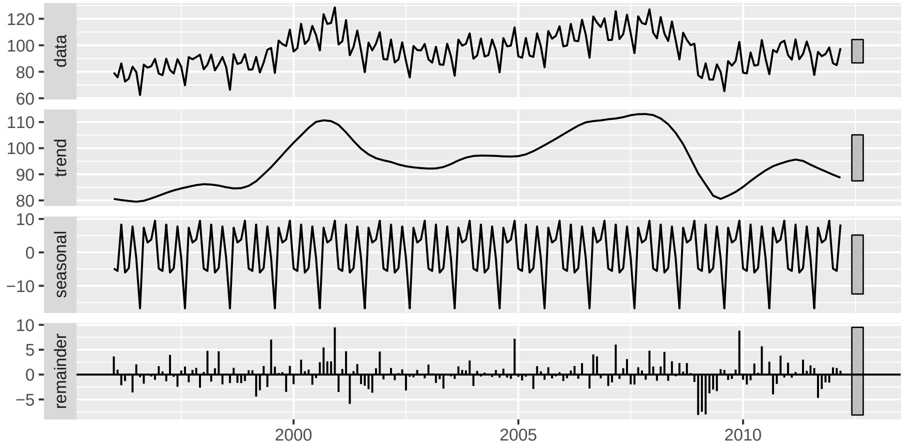
```
elecequip %>%
  stl(t.window=15, s.window='periodic', robust=TRUE) %>%
  autoplot
```



STL decomposition in R

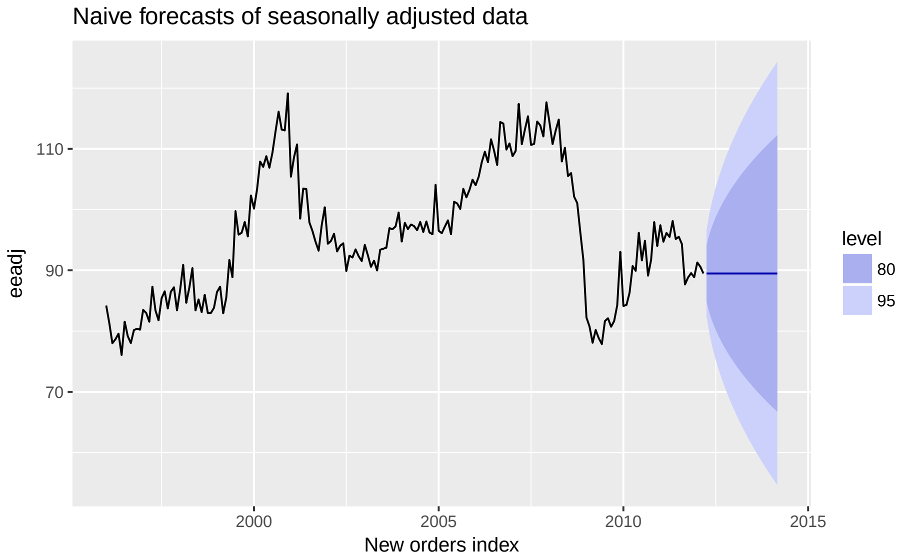
- `t.window` controls wiggliness of trend component.
- `s.window` controls variation on seasonal component.

Outline

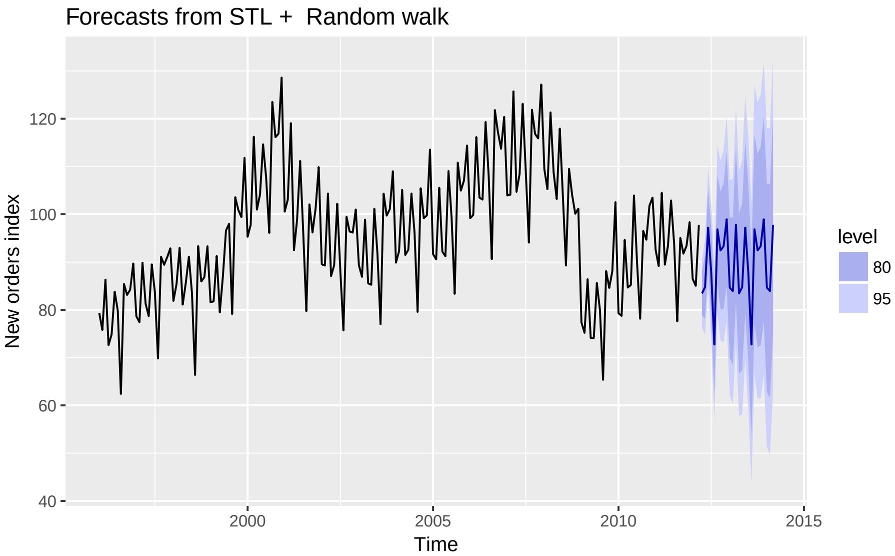
Forecasting and decomposition

- Forecast seasonal component by repeating the last year
- Forecast seasonally adjusted data using non-seasonal time series method. E.g.,
  - Holt's method — next topic
  - Random walk with drift model
- Combine forecasts of seasonal component with forecasts of seasonally adjusted data to get forecasts of original data.
- Sometimes a decomposition is useful just for understanding the data before building a separate forecasting model.

Seas adj elec equipment



Seas adj elec equipment



How to do this in R

```
fit <- stl(elecequip, t.window=15,
  s.window="periodic", robust=TRUE)

eeadj <- seasadj(fit)
autoplot(naive(eeadj, h=24)) +
  ylab("New orders index")

fcast <- forecast(fit, method="naive", h=24)
autoplot(fcast) +
  ylab="New orders index")
```

Decomposition and prediction intervals

- It is common to take the prediction intervals from the seasonally adjusted forecasts and modify them with the seasonal component.
- This ignores the uncertainty in the seasonal component estimate.
- It also ignores the uncertainty in the future seasonal pattern.

Some more R functions

```r
fcast <- stlf(elecequip, method='naive')

fcast <- stlf(elecequip, method='naive',
  h=36, s.window=11, robust=TRUE)
```

Outline

Lab Session 5