1.3 Seasonality and trends
Outline

1. Time series components
2. STL decomposition
3. Forecasting and decomposition
4. Lab session 5

Forecasting: principles and practice
Time series patterns

Trend pattern exists when there is a long-term increase or decrease in the data.

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Cyclic pattern exists when data exhibit rises and falls that are not of fixed period (duration usually of at least 2 years).
Time series decomposition

\[ Y_t = f(S_t, T_t, E_t) \]

where

- \( Y_t = \) data at period \( t \)
- \( S_t = \) seasonal component at period \( t \)
- \( T_t = \) trend-cycle component at period \( t \)
- \( E_t = \) remainder (or irregular or error) component at period \( t \)

Additive decomposition: \( Y_t = S_t + T_t + E_t \).

Multiplicative decomposition: \( Y_t = S_t \times T_t \times E_t \).
Time series decomposition

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where

- \( Y_t \) = data at period \( t \)
- \( S_t \) = seasonal component at period \( t \)
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**Additive decomposition:** \( Y_t = S_t + T_t + E_t \).

**Multiplicative decomposition:** \( Y_t = S_t \times T_t \times E_t \).
Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.

If seasonal are proportional to level of series, then multiplicative model appropriate.

Multiplicative decomposition more prevalent with economic series

Alternative: use a Box-Cox transformation, and then use additive decomposition.

Logs turn multiplicative relationship into an additive relationship:

\[ Y_t = S_t \times T_t \times E_t \quad \Rightarrow \quad \log Y_t = \log S_t + \log T_t + \log E_t. \]
Euro electrical equipment

Electrical equipment manufacturing (Euro area)

New orders index

Data

Trend

Forecasting: principles and practice
Euro electrical equipment

Data trend seasonal remainder

2000 2005 2010
60 80 100 120
80 90 100 110
−20 −10 0 10
−8 −4 0 4

Time series components
Euro electrical equipment
Useful by-product of decomposition: an easy way to calculate seasonally adjusted data.

Additive decomposition: seasonally adjusted data given by

$$Y_t - S_t = T_t + E_t$$

Multiplicative decomposition: seasonally adjusted data given by

$$Y_t / S_t = T_t \times E_t$$
Euro electrical equipment

Electrical equipment manufacturing

New orders index

- Data
- SeasAdjust

Forecasting: principles and practice

Time series components
Seasonal adjustment

- We use estimates of $S$ based on past values to seasonally adjust a current value.
- Seasonally adjusted series reflect **remainders** as well as **trend**. Therefore they are not “smooth” and “downturns” or “upturns” can be misleading.
- It is better to use the trend-cycle component to look for turning points.
- Classical method originated in 1920s.
- Census II method introduced in 1957. Basis for modern X-12-ARIMA method.
- STL method introduced in 1983
- TRAMO/SEATS introduced in 1990s.
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STL decomposition

- STL: “Seasonal and Trend decomposition using Loess”,
- Very versatile and robust.
- Unlike X-12-ARIMA, STL will handle any type of seasonality.
- Seasonal component allowed to change over time, and rate of change controlled by user.
- Smoothness of trend-cycle also controlled by user.
- Robust to outliers
- Not trading day or calendar adjustments.
- Only additive.
elecequip %>% stl(s.window=5) %>% autoplot
Euro electrical equipment

elecequip %>%
  stl(t.window=15, s.window='periodic', robust=TRUE) %>%
  autoplot

Forecasting: principles and practice
STL decomposition

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t.window controls wiggliness of trend component.

s.window controls variation on seasonal component.
1 Time series components

2 STL decomposition

3 Forecasting and decomposition

4 Lab session 5
Forecasting and decomposition

- Forecast seasonal component by repeating the last year
- Forecast seasonally adjusted data using non-seasonal time series method. E.g.,
  - Holt’s method — next topic
  - Random walk with drift model
- Combine forecasts of seasonal component with forecasts of seasonally adjusted data to get forecasts of original data.
- Sometimes a decomposition is useful just for understanding the data before building a separate forecasting model.
Naive forecasts of seasonally adjusted data

Forecasting: principles and practice
Forecasting and decomposition
Seas adj elec equipment

Forecasting: principles and practice

Forecasting and decomposition

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fit <- `stl`(elecequip, t.window=15,
  s.window="periodic", robust=TRUE)

eeadj <- `seasadj`(fit)
autoplot(`naive`(eeadj, h=24)) +
ylab("New orders index")

fcast <- `forecast`(fit, method="naive", h=24)
autoplot(fcast) +
ylab="New orders index")
Decomposition and prediction intervals

- It is common to take the prediction intervals from the seasonally adjusted forecasts and modify them with the seasonal component.
- This ignores the uncertainty in the seasonal component estimate.
- It also ignores the uncertainty in the future seasonal pattern.
Some more R functions

\begin{verbatim}
fcast <- stlf(elecequip, method='naive')
fcast <- stlf(elecequip, method='naive', h=36, s.window=11, robust=TRUE)
\end{verbatim}
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Lab Session 5