MONASH University

MONASH BUSINESS SCHOOL

# Forecasting: principles and practice

**Rob J Hyndman**

## 1.1 Time series graphics

# Outline

# ts objects and ts function

A time series is stored in a ts object in R:

- a list of numbers
- information about times those numbers were recorded.

## Example

| Year | Observation |
|---|---|
| 2012 | 123 |
| 2013 | 39 |
| 2014 | 78 |
| 2015 | 52 |
| 2016 | 110 |

```r
y <- ts(c(123,39,78,52,110), start=2012)
```

# `ts` objects and `ts` function

For observations that are more frequent than once per year, add a `frequency` argument.

E.g., monthly data stored as a numerical vector `z`:

```
y <- ts(z, frequency=12, start=c(2003, 1))
```

# `ts` objects and `ts` function

`ts(data, frequency, start)`

| Type of data | frequency | start example |
|---|---|---|
| Annual | | |
| Quarterly | | |
| Monthly | | |
| Daily | | |
| Weekly | | |
| Hourly | | |
| Half-hourly | | |

# `ts` objects and `ts` function

**`ts(data, frequency, start)`**

| Type of data | frequency | start example |
|---|---|---|
| Annual | 1 | |
| Quarterly | | |
| Monthly | | |
| Daily | | |
| Weekly | | |
| Hourly | | |
| Half-hourly | | |

# `ts` objects and `ts` function

`ts(data, frequency, start)`

| Type of data | frequency | start example |
|---|---|---|
| Annual | 1 | 1995 |
| Quarterly | | |
| Monthly | | |
| Daily | | |
| Weekly | | |
| Hourly | | |
| Half-hourly | | |

# `ts` objects and `ts` function

`ts(data, frequency, start)`

| Type of data | frequency | start example |
|---|---|---|
| Annual | 1 | 1995 |
| Quarterly | 4 | |
| Monthly | | |
| Daily | | |
| Weekly | | |
| Hourly | | |
| Half-hourly | | |

# `ts` objects and `ts` function

`ts(data, frequency, start)`

| Type of data | frequency | start example |
|---|---|---|
| Annual | 1 | 1995 |
| Quarterly | 4 | c(1995,2) |
| Monthly | | |
| Daily | | |
| Weekly | | |
| Hourly | | |
| Half-hourly | | |

# `ts` objects and `ts` function

**`ts(data, frequency, start)`**

| Type of data | frequency | start example |
|---|---|---|
| Annual | 1 | 1995 |
| Quarterly | 4 | c(1995,2) |
| Monthly | 12 | |
| Daily | | |
| Weekly | | |
| Hourly | | |
| Half-hourly | | |

# `ts` objects and `ts` function

```
ts(data, frequency, start)
```

| Type of data | frequency | start example |
|---|---|---|
| Annual | 1 | 1995 |
| Quarterly | 4 | c(1995,2) |
| Monthly | 12 | c(1995,9) |
| Daily | | |
| Weekly | | |
| Hourly | | |
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```
ts(data, frequency, start)
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| Type of data | frequency | start example |
|---|---|---|
| Annual | 1 | 1995 |
| Quarterly | 4 | c(1995,2) |
| Monthly | 12 | c(1995,9) |
| Daily | 7 *or* 365.25 | |
| Weekly | | |
| Hourly | | |
| Half-hourly | | |

# `ts` objects and `ts` function

`ts(data, frequency, start)`

| Type of data | frequency | start example |
|---|---:|---|
| Annual | 1 | 1995 |
| Quarterly | 4 | c(1995,2) |
| Monthly | 12 | c(1995,9) |
| Daily | 7 *or* 365.25 | 1 *or* c(1995,234) |
| Weekly | | |
| Hourly | | |
| Half-hourly | | |

# `ts` objects and `ts` function

`ts(data, frequency, start)`

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| Weekly | 52.18 | c(1995,23) |
| Hourly | 24 *or* 168 *or* 8,766 | |
| Half-hourly | | |

# `ts` objects and `ts` function

`ts(data, frequency, start)`

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| Hourly | 24 *or* 168 *or* 8,766 | 1 |
| Half-hourly | 48 *or* 336 *or* 17,532 | |

# `ts` objects and `ts` function

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|---|---:|---|
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| Quarterly | 4 | c(1995,2) |
| Monthly | 12 | c(1995,9) |
| Daily | 7 *or* 365.25 | 1 *or* c(1995,234) |
| Weekly | 52.18 | c(1995,23) |
| Hourly | 24 *or* 168 *or* 8,766 | 1 |
| Half-hourly | 48 *or* 336 *or* 17,532 | 1 |

# Australian GDP

```
ausgdp <- ts(x, frequency=4, start=c(1971,3))
```

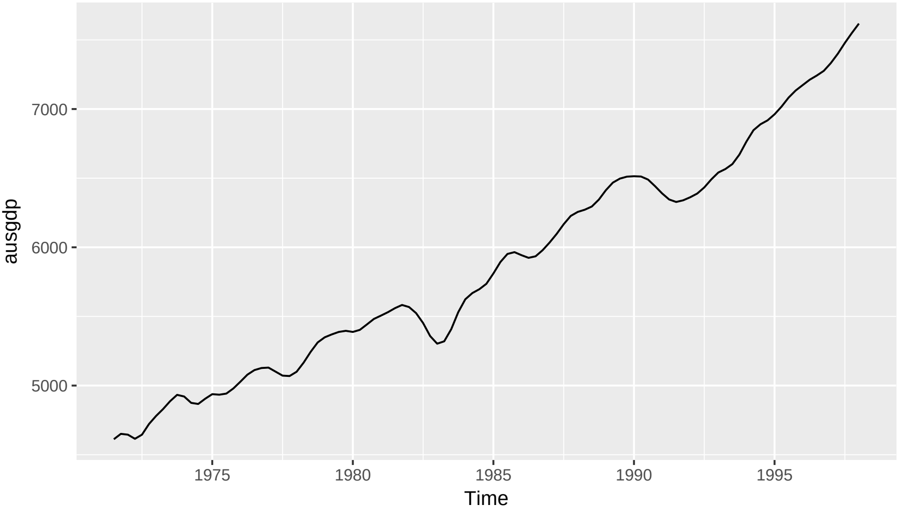
- Class: "ts"
- Print and plotting methods available.

```
ausgdp
```

```
##      Qtr1 Qtr2 Qtr3 Qtr4
## 1971           4612 4651
## 1972 4645 4615 4645 4722
## 1973 4780 4830 4887 4933
## 1974 4921 4875 4867 4905
## 1975 4938 4934 4942 4979
## 1976 5028 5079 5112 5127
```

# Australian GDP

```r
autoplot(ausgdp)
```



# Residential electricity sales

```
elecsales
```

```
## Time Series:
## Start = 1989
## End = 2008
## Frequency = 1
##  [1] 2354.34 2379.71 2318.52 2468.99 2386.09
##  [9] 2844.50 3000.70 3108.10 3357.50 3075.70
## [17] 3430.60 3527.48 3637.89 3655.00
```

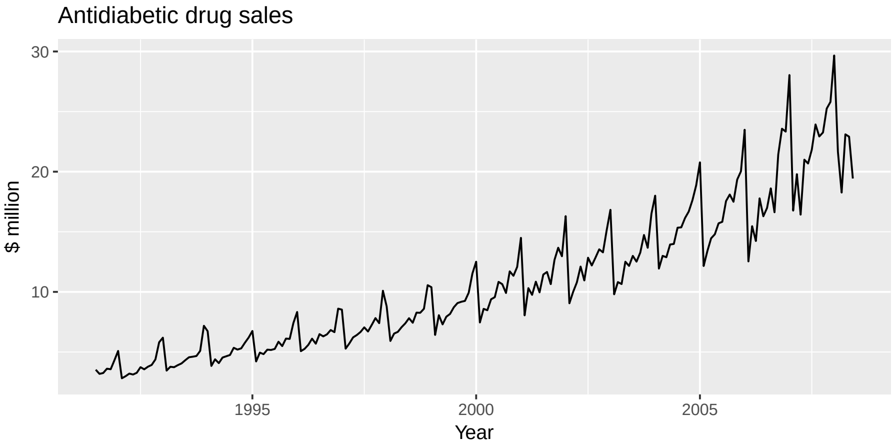
# Outline

# Time plots

```
autoplot(melsyd[,"Economy.Class"])
```

# Time plots

```r
autoplot(a10) + ylab("$ million") + xlab("Year") +
         ggtitle("Antidiabetic drug sales")
```



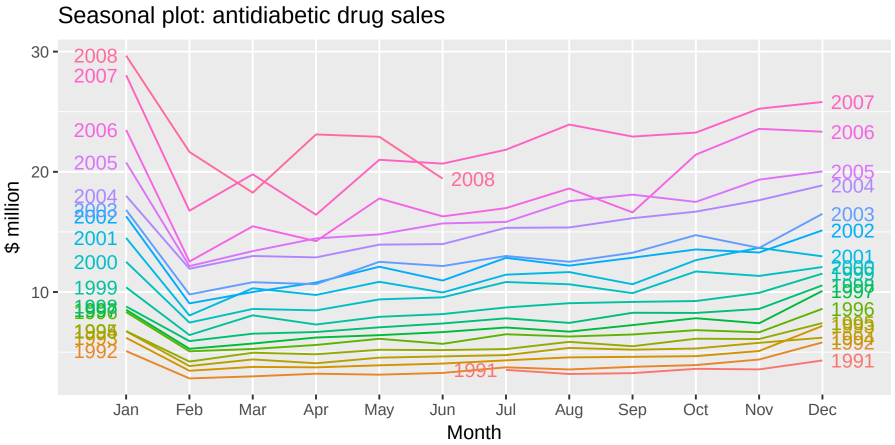
# Outline

# Lab Session 1

# Outline

# Seasonal plots

```r
ggseasonplot(a10, ylab="$ million",
  year.labels=TRUE, year.labels.left=TRUE) +
  ggtitle("Seasonal plot: antidiabetic drug sales")
```

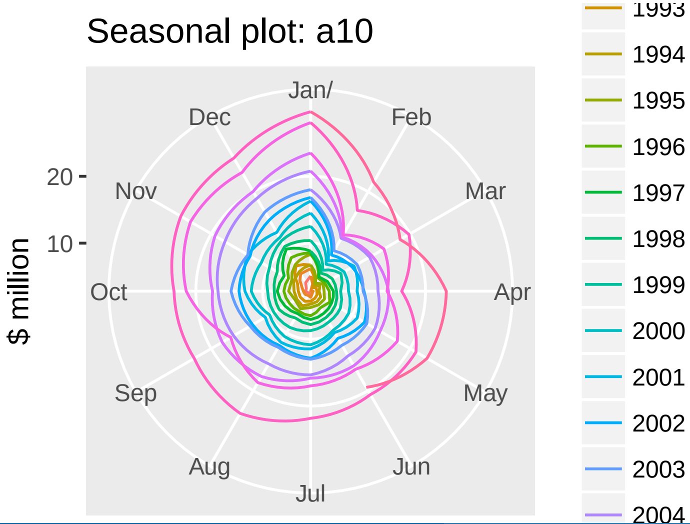


# Seasonal plots

- Data plotted against the individual “seasons” in which the data were observed. (In this case a “season” is a month.)
- Something like a time plot except that the data from each season are overlapped.
- Enables the underlying seasonal pattern to be seen more clearly, and also allows any substantial departures from the seasonal pattern to be easily identified.
- In R: `ggseasonplot`

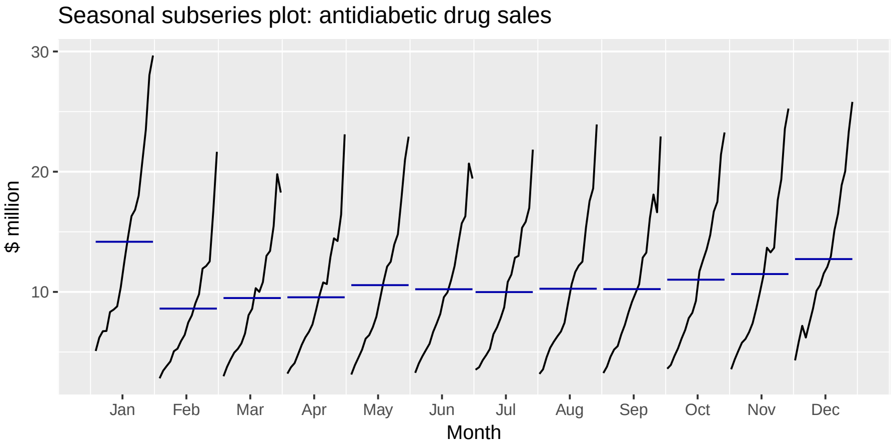
# Seasonal polar plots

```
ggseasonplot(a10, polar=TRUE) + ylab("$ million")
```



# Seasonal subseries plots

```
ggsubseriesplot(a10) + ylab("$ million") +
  ggtitle("Seasonal subseries plot: antidiabetic drug sal
```

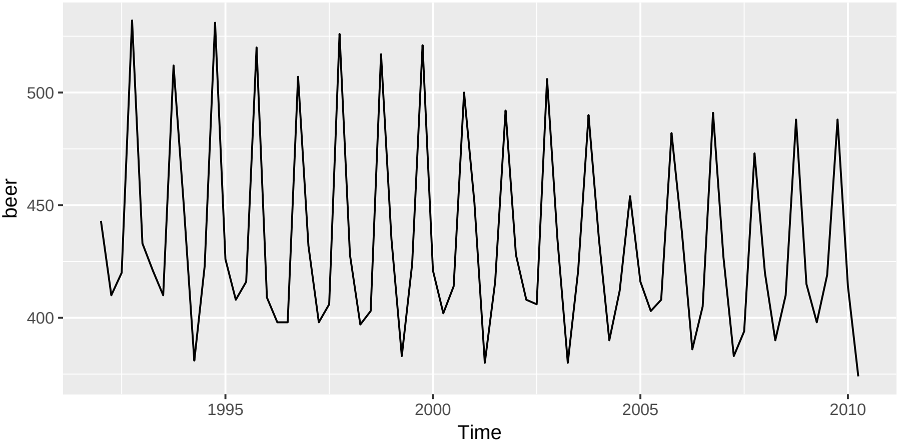


# Seasonal subseries plots

- Data for each season collected together in time plot as separate time series.
- Enables the underlying seasonal pattern to be seen clearly, and changes in seasonality over time to be visualized.
- In R: `ggsubseriesplot`

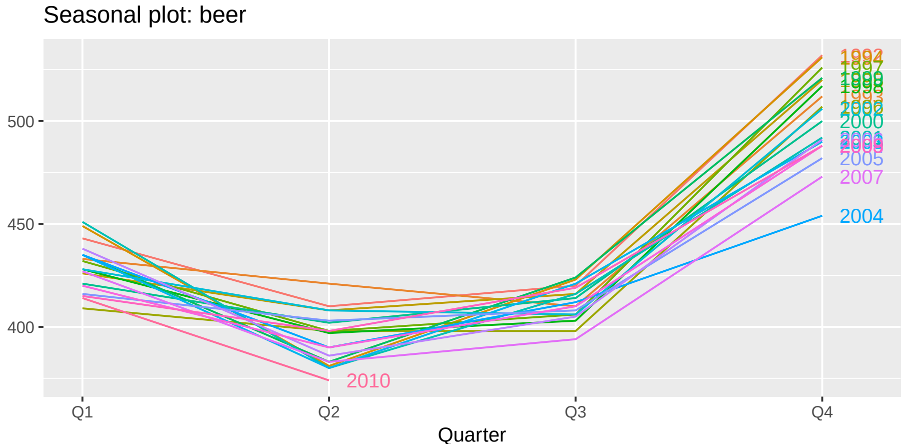
# Quarterly Australian Beer Production

```r
beer <- window(ausbeer,start=1992)
autoplot(beer)
```



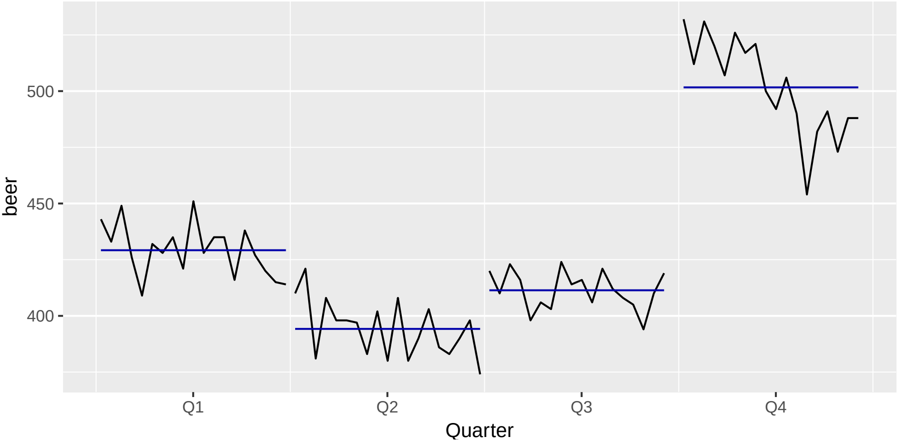
# Quarterly Australian Beer Production

```r
ggseasonplot(beer,year.labels=TRUE)
```



# Quarterly Australian Beer Production

```
ggsubseriesplot(beer)
```



# Outline

# Time series patterns

**Trend** pattern exists when there is a long-term increase or decrease in the data.

**Seasonal** pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

**Cyclic** pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

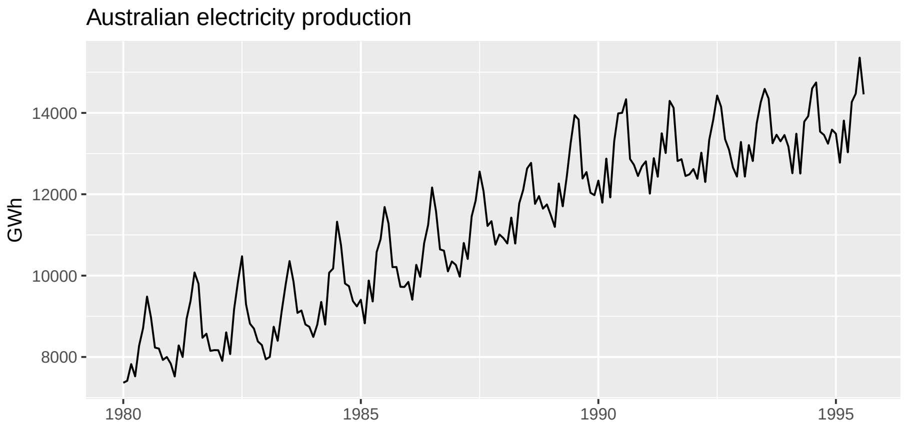
# Time series components

## Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

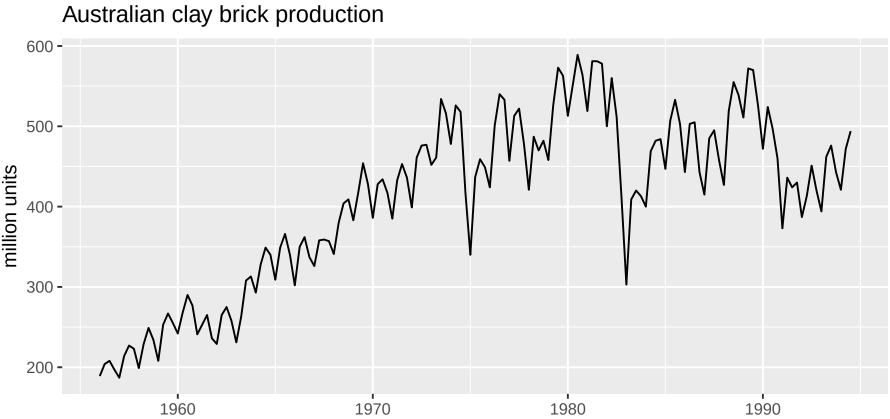
# Time series patterns

```r
autoplot(window(elec, start=1980)) +
  ggtitle("Australian electricity production") +
  xlab("Year") + ylab("GWh")
```



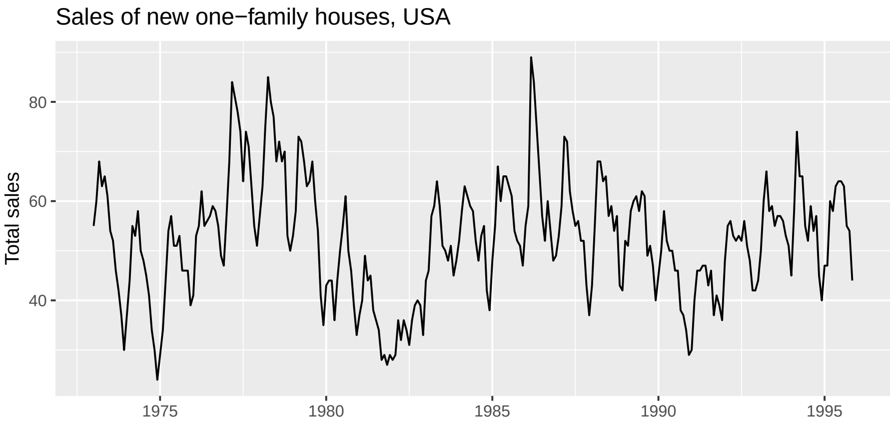
# Time series patterns

```r
autoplot(bricksq) +
  ggtitle("Australian clay brick production") +
  xlab("Year") + ylab("million units")
```



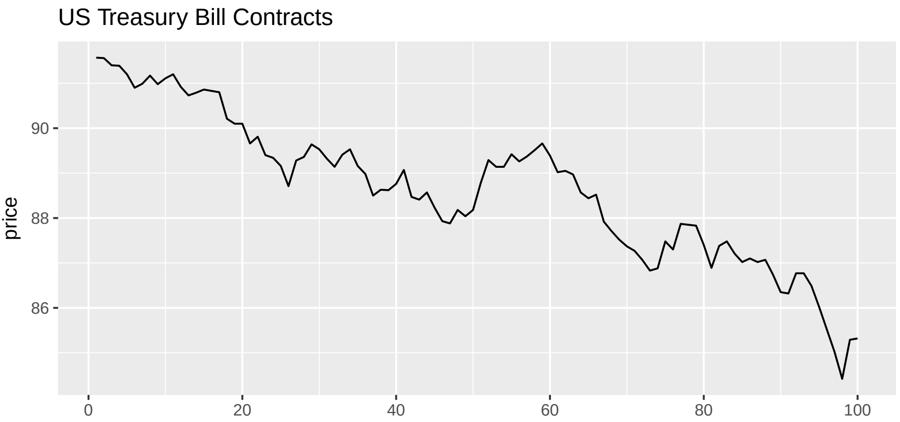
# Time series patterns

```r
autoplot(hsales) +
  ggtitle("Sales of new one-family houses, USA")
  xlab("Year") + ylab("Total sales")
```



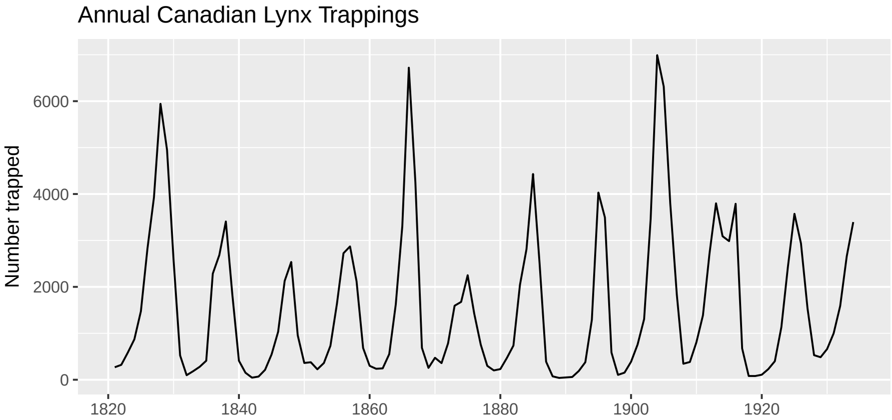
# Time series patterns

```r
autoplot(ustreas) +
  ggtitle("US Treasury Bill Contracts") +
  xlab("Day") + ylab("price")
```



# Time series patterns

```r
autoplot(lynx) +
  ggtitle("Annual Canadian Lynx Trappings") +
  xlab("Year") + ylab("Number trapped")
```



# Seasonal or cyclic?

Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
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- magnitude of cycle more variable than magnitude of seasonal pattern

The timing of peaks and troughs is predictable with seasonal data, but unpredictable in the long term with cyclic data.

# Seasonal or cyclic?

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- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

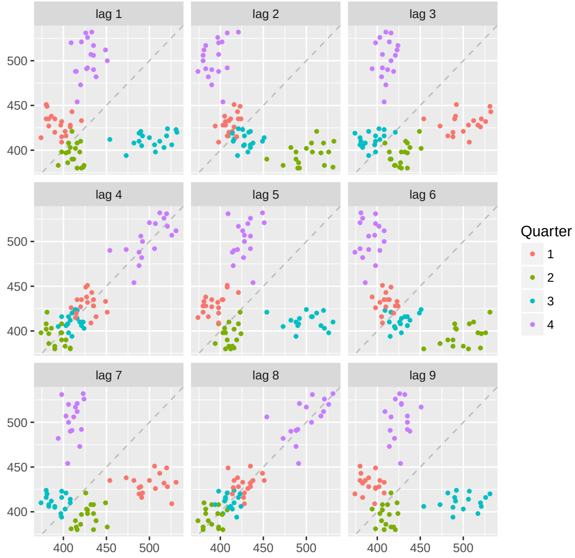
The timing of peaks and troughs is predictable with seasonal data, but unpredictable in the long term with cyclic data.

# Outline

# Example: Beer production

```r
beer <- window(ausbeer, start=1992)
gglagplot(beer, lags=9, do.lines=FALSE,
  continuous=FALSE)
```

# Example: Beer production



# Lagged scatterplots

- Each graph shows $y_t$ plotted against $y_{t-k}$ for different values of $k$.
- The autocorrelations are the correlations associated with these scatterplots.

# Autocorrelation

**Covariance** and **correlation**: measure extent of **linear relationship** between two variables ($y$ and $X$).

**Autocovariance** and **autocorrelation**: measure linear relationship between **lagged values** of a time series $y$.

We measure the relationship between: $y_t$ and $y_{t-1}$
$y_t$ and $y_{t-2}$
$y_t$ and $y_{t-3}$
etc.

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We measure the relationship between: $y_t$ and $y_{t-1}$
$y_t$ and $y_{t-2}$
$y_t$ and $y_{t-3}$
etc.

# Autocorrelation

We denote the sample autocovariance at lag $k$ by $c_k$ and the sample autocorrelation at lag $k$ by $r_k$. Then define

$$c_k = \frac{1}{T}\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})$$

and $$r_k = c_k/c_0$$

- $r_1$ indicates how successive values of $y$ relate to each other
- $r_2$ indicates how $y$ values two periods apart relate to each other
- $r_k$ is *almost* the same as the sample correlation between $y_t$ and $y_{t-k}$.

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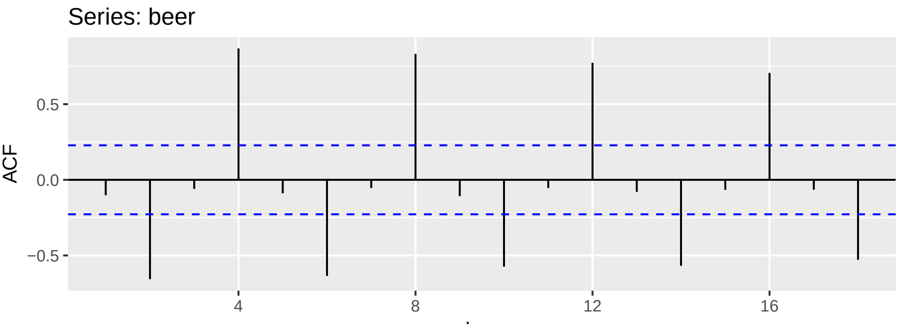
# Autocorrelation

Results for first 9 lags for beer data:/footnotesize

| $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ | $r_6$ | $r_7$ | $r_8$ | $r$ |
|---|---|---|---|---|---|---|---|---|
| -0.102 | -0.657 | -0.060 | 0.869 | -0.089 | -0.635 | -0.054 | 0.832 | -0. |

```r
ggAcf(beer)
```



# Autocorrelation

- $r_4$ higher than for the other lags. This is due to **the seasonal pattern in the data**: the peaks tend to be **4 quarters** apart and the troughs tend to be **2 quarters** apart.
- $r_2$ is more negative than for the other lags because troughs tend to be 2 quarters behind peaks.
- Together, the autocorrelations at lags 1, 2, …, make up the *autocorrelation* or ACF.
- The plot is known as a **correlogram**

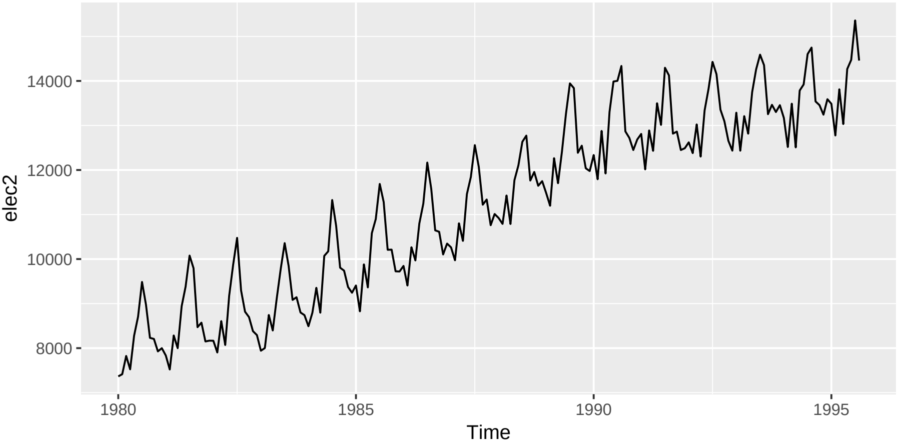
# Recognizing seasonality in a time series

If there is seasonality, the ACF at the seasonal lag (e.g., 12 for monthly data) will be **large and positive**.

- For seasonal monthly data, a large ACF value will be seen at lag 12 and possibly also at lags 24, 36, . . .
- For seasonal quarterly data, a large ACF value will be seen at lag 4 and possibly also at lags 8, 12, . . .

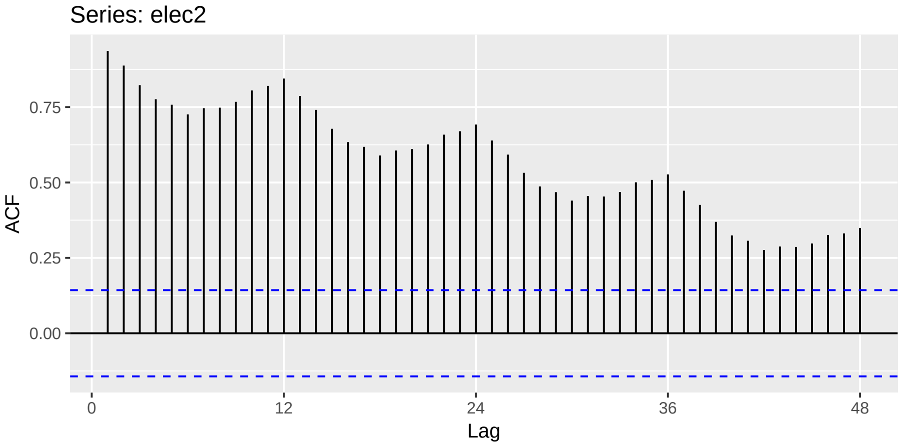
# Aus monthly electricity production

```r
elec2 <- window(elec, start=1980)
autoplot(elec2)
```



# Aus monthly electricity production

```r
ggAcf(elec2, lag.max=48)
```



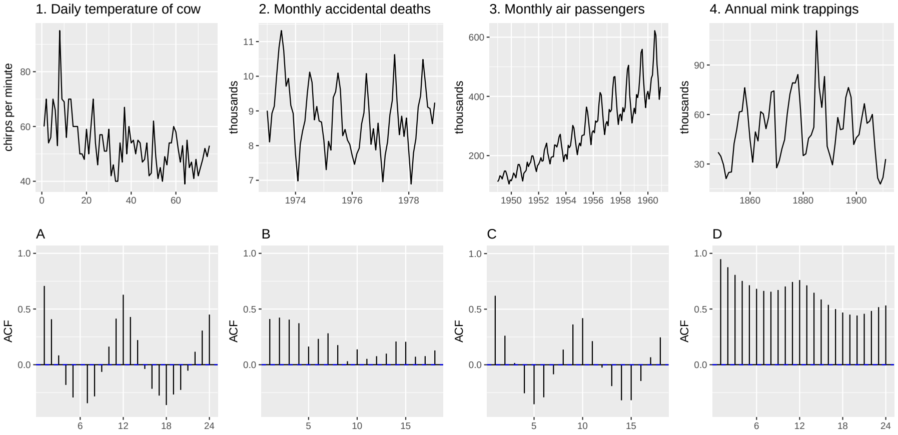
# Aus monthly electricity production

Time plot shows clear trend and seasonality.

The same features are reflected in the ACF.

- The slowly decaying ACF indicates trend.
- The ACF peaks at lags 12, 24, 36, …, indicate seasonality of length 12.

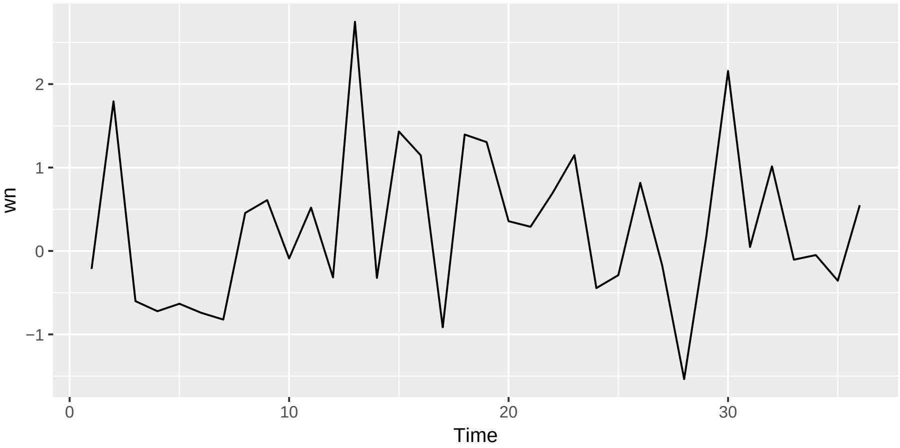
# Which is which?



# Outline

# Example: White noise

```
wn <- ts(rnorm(36))
autoplot(wn)
```



# Example: White noise

```r
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autoplot(wn)
```
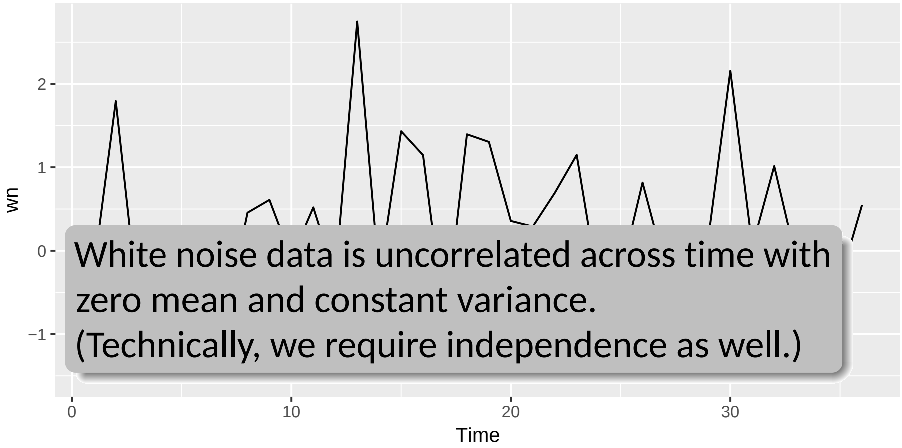
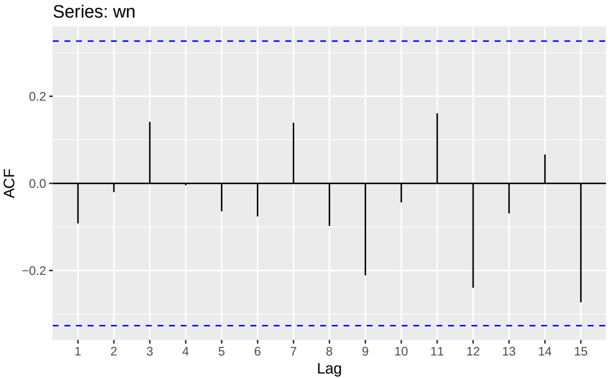




White noise data is uncorrelated across time with zero mean and constant variance.
(Technically, we require independence as well.)

# Example: White noise

| | |
|---|---|
| $r_1$ | -0.09 |
| $r_2$ | -0.02 |
| $r_3$ | 0.14 |
| $r_4$ | -0.00 |
| $r_5$ | -0.06 |
| $r_6$ | -0.08 |
| $r_7$ | 0.14 |
| $r_8$ | -0.10 |
| $r_9$ | -0.21 |
| $r_{10}$ | -0.04 |




Sample autocorrelations for white noise series.

For uncorrelated data, we would expect each autocorrelation to be close to zero.

# Sampling distribution of autocorrelations

Sampling distribution of $r_k$ for white noise data is asymptotically N$(0,1/T)$.

- 95% of all $r_k$ for white noise must lie within $\pm 1.96/\sqrt{T}$.
- If this is not the case, the series is probably not WN.
- Common to plot lines at $\pm 1.96/\sqrt{T}$ when plotting ACF. These are the ***critical values***.

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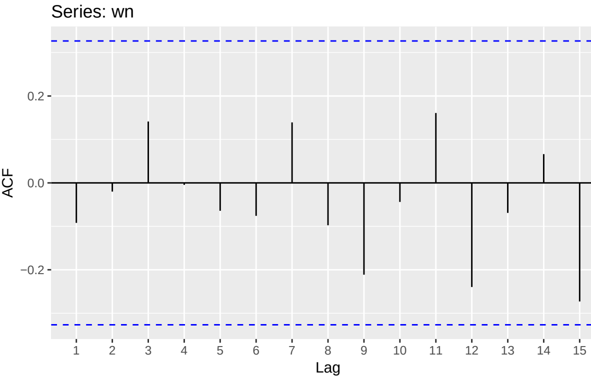
# Autocorrelation

## Example:

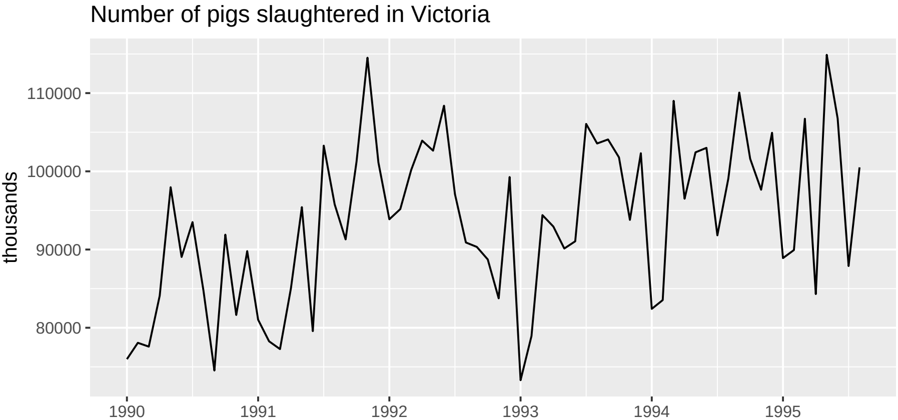
$T = 36$ and so critical values at $\pm 1.96/\sqrt{36} = \pm 0.327$.

All autocorrelation coefficients lie within these limits, confirming that the data are white noise. (More precisely, the data cannot be distinguished from white noise.)



# Example: Pigs slaughtered

```r
pigs2 <- window(pigs, start=1990)
autoplot(pigs2) +
  xlab("Year") + ylab("thousands") +
  ggtitle("Number of pigs slaughtered in Victoria")
```



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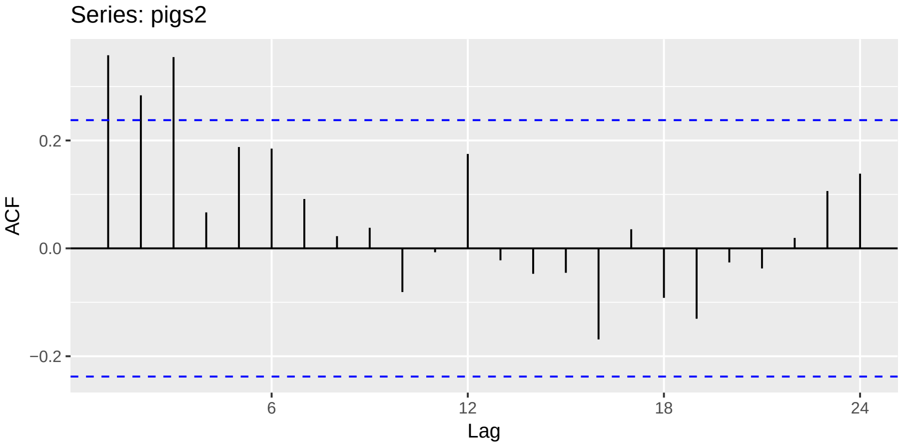
```



# Example: Pigs slaughtered

Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 1990 through August 1995. (Source: Australian Bureau of Statistics.)

- Difficult to detect pattern in time plot.
- ACF shows some significant autocorrelation at lags 1, 2, and 3.
- $r_{12}$ relatively large although not significant. This may indicate some slight seasonality.

These show the series is **not a white noise series.**

# Example: Pigs slaughtered

Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 1990 through August 1995. (Source: Australian Bureau of Statistics.)

- Difficult to detect pattern in time plot.
- ACF shows some significant autocorrelation at lags 1, 2, and 3.
- $r_{12}$ relatively large although not significant. This may indicate some slight seasonality.

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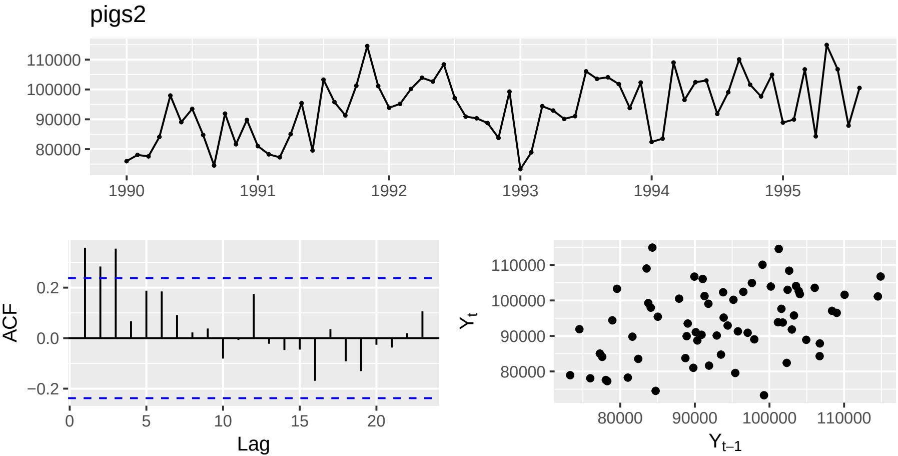
# Combination graph

```r
ggtsdisplay(pigs2, plot.type='scatter')
```



# Outline

# Lab Session 2