MONASH University
REVOLUTION ANALYTICS
Rob J Hyndman

Forecasting using

3. Autocorrelation and seasonality

OTexts.com/fpp/2/
OTexts.com/fpp/6/1

Outline

Time series graphics

- **Time plots**

  R command: `plot` or `plot.ts`
- **Seasonal plots**

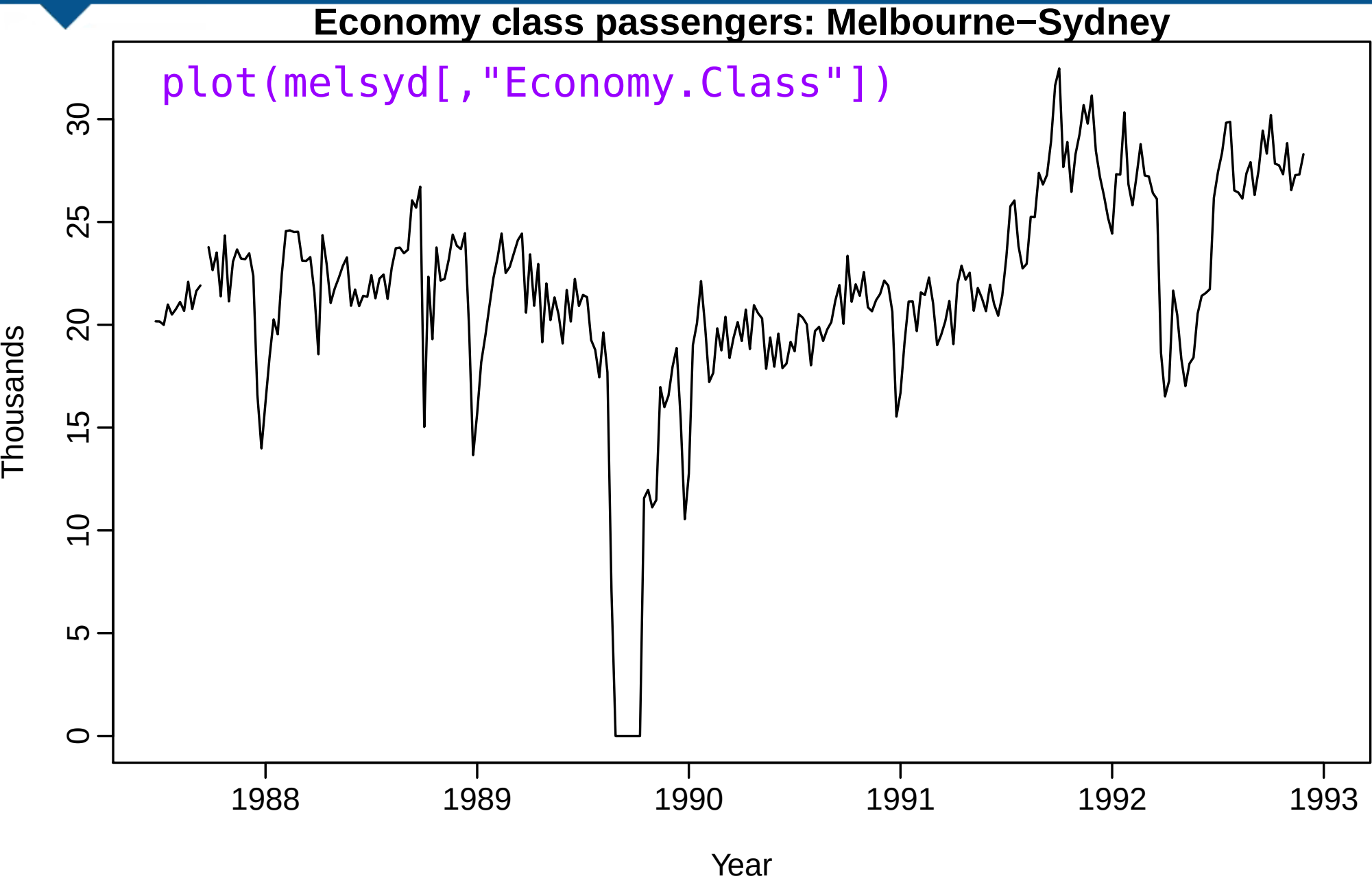
  R command: `seasonplot`
- **Seasonal subseries plots**

  R command: `monthplot`
- **Lag plots**

  R command: `lag.plot`
- **ACF plots**

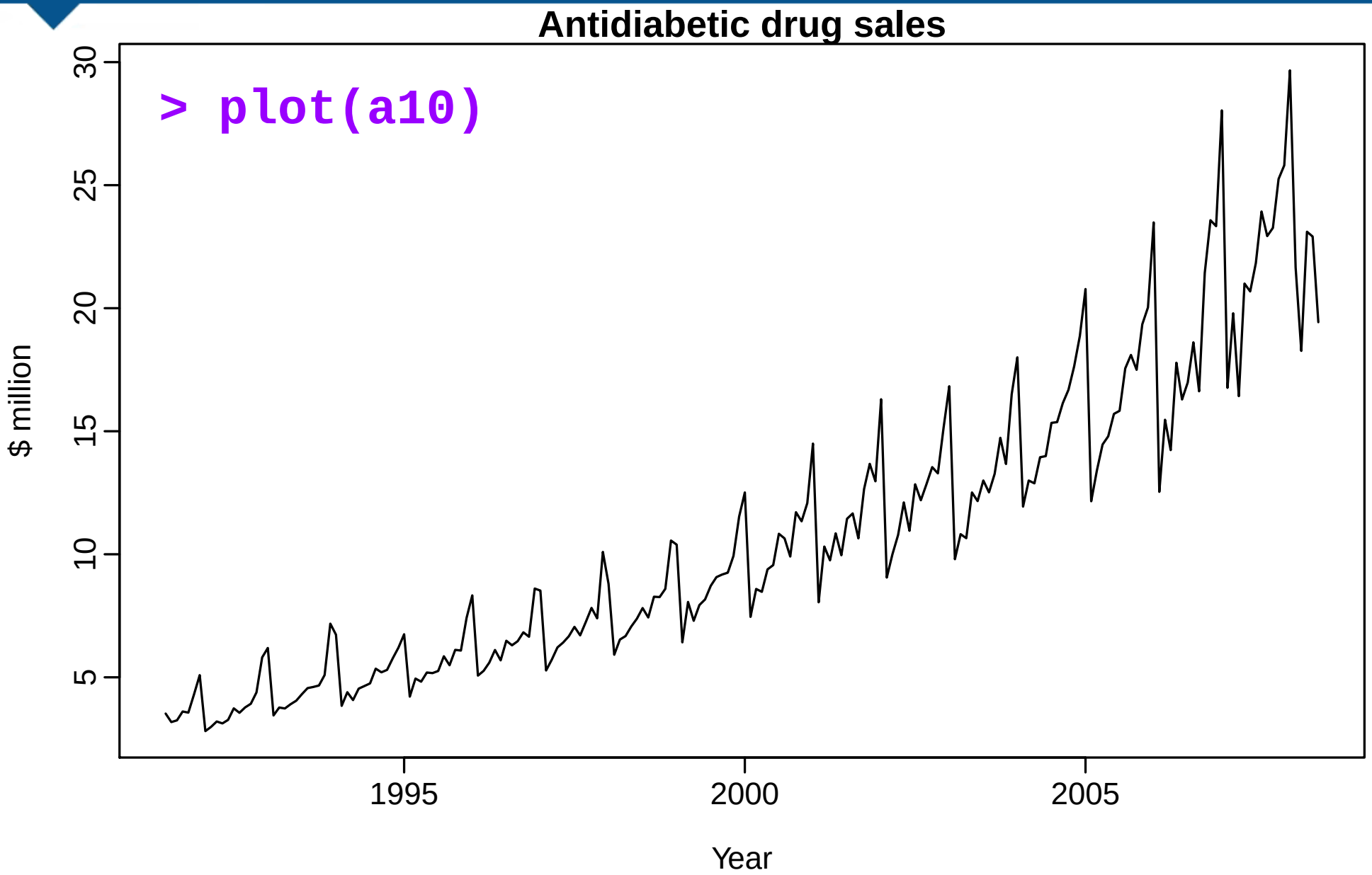
  R command: `Acf`

Time series graphics



Time series graphics

```
> plot(a10)
```



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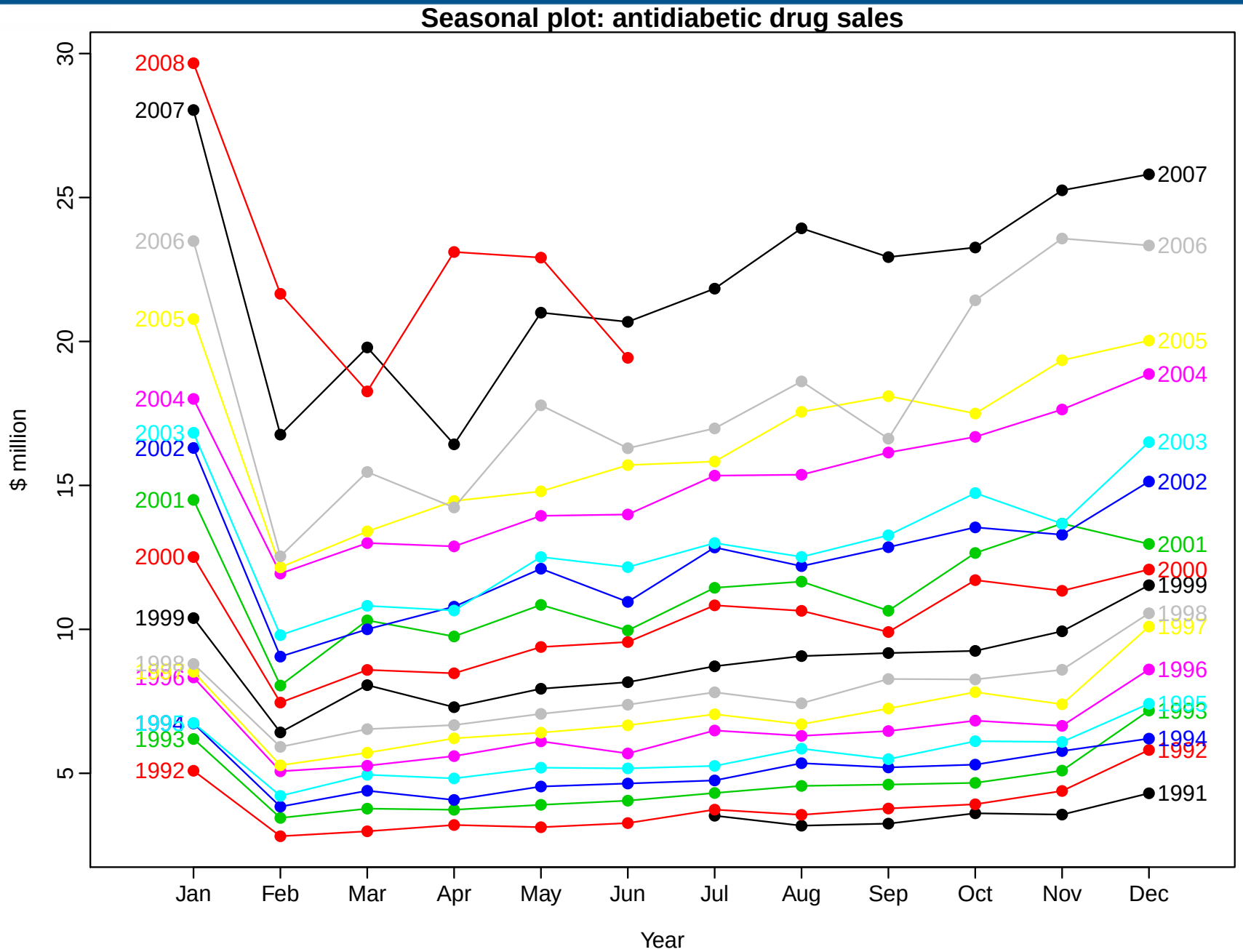

Seasonal plots

- Data plotted against the individual “seasons” in which the data were observed. (In this case a “season” is a month.)
- Something like a time plot except that the data from each season are overlapped.
- Enables the underlying seasonal pattern to be seen more clearly, and also allows any substantial departures from the seasonal pattern to be easily identified.
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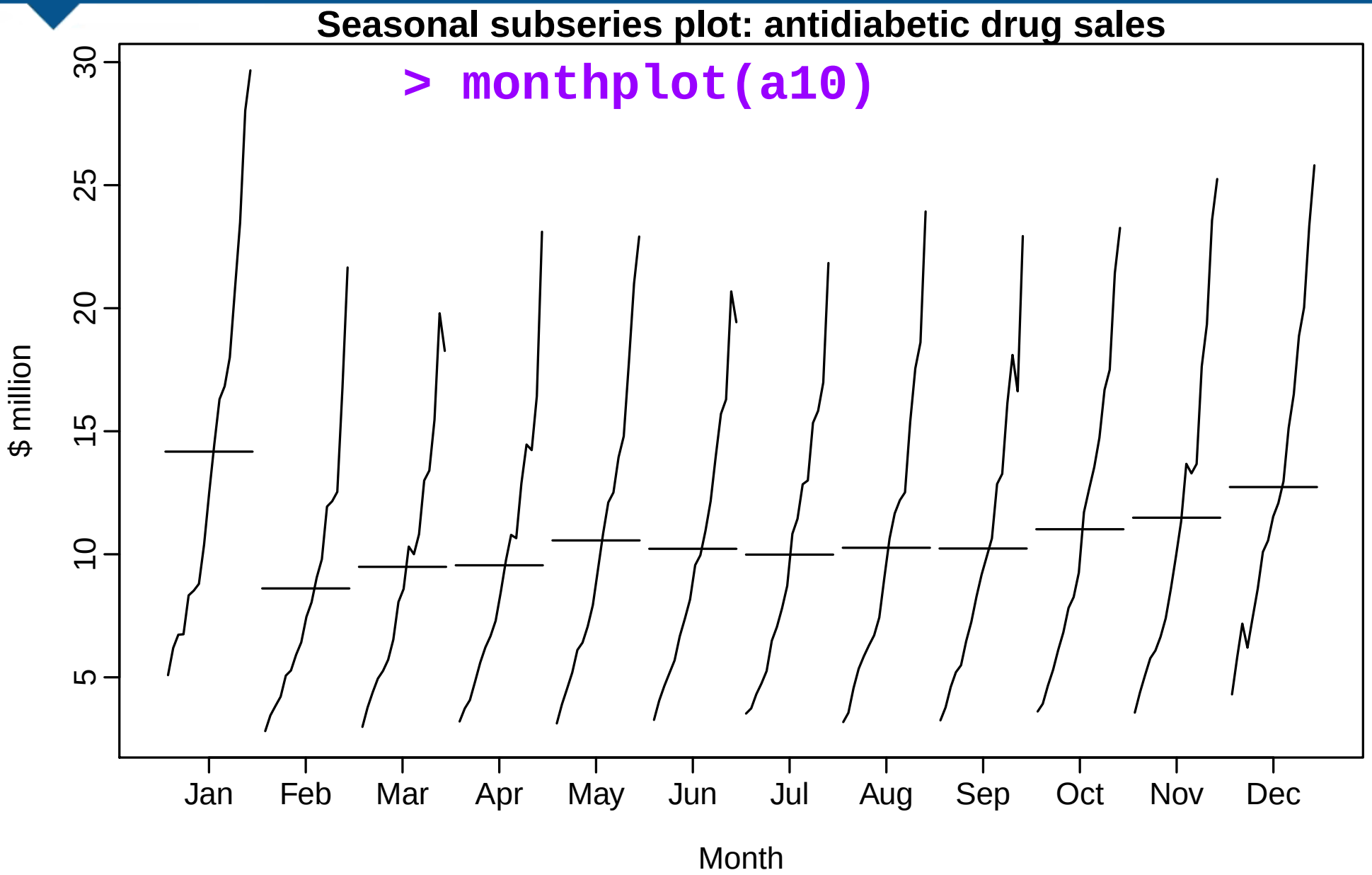

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Quarterly Australian Beer Production

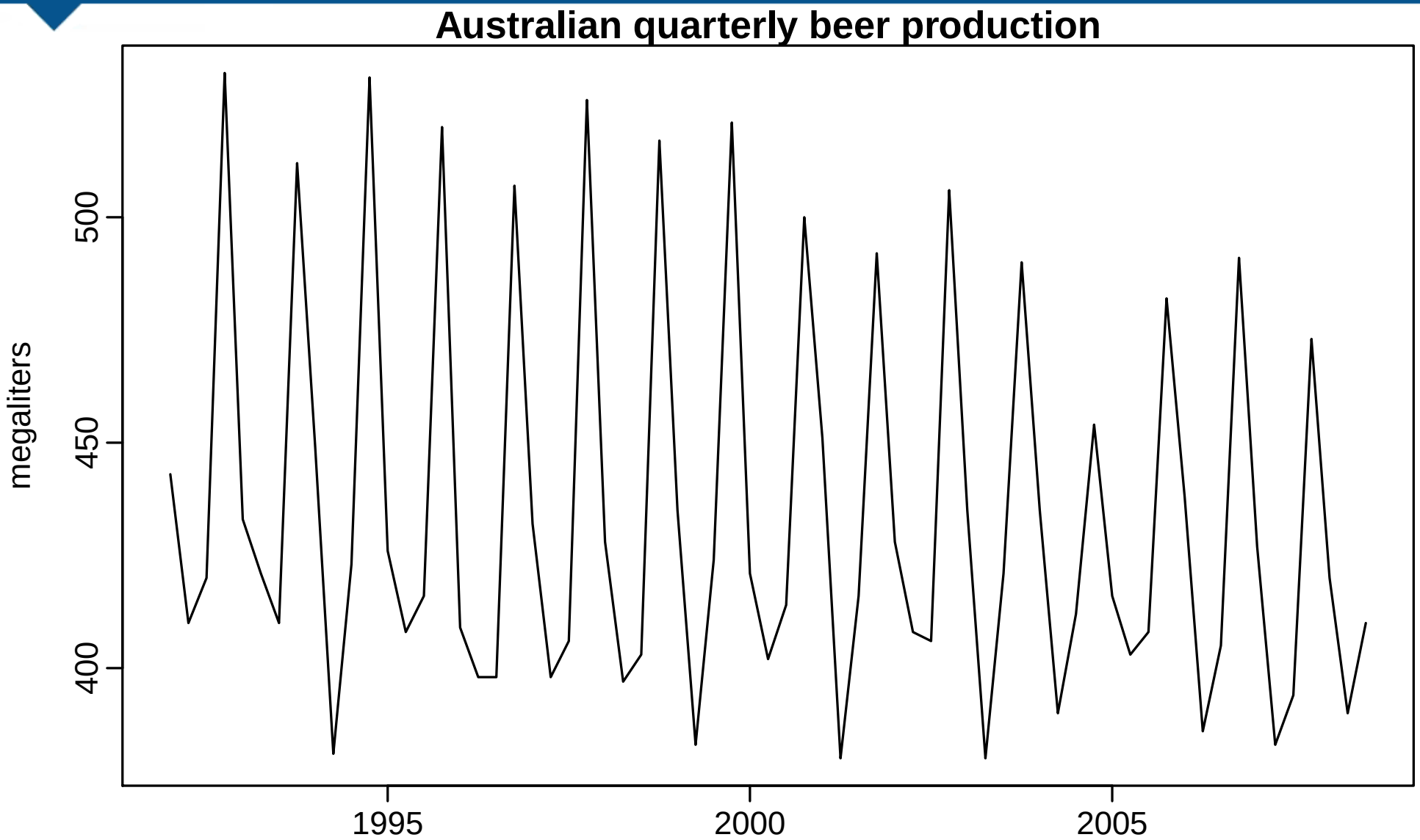
```
beer <- window(ausbeer,start=1992)

plot(beer)

seasonplot(beer,year.labels=TRUE)

monthplot(beer)
```

Time series graphics



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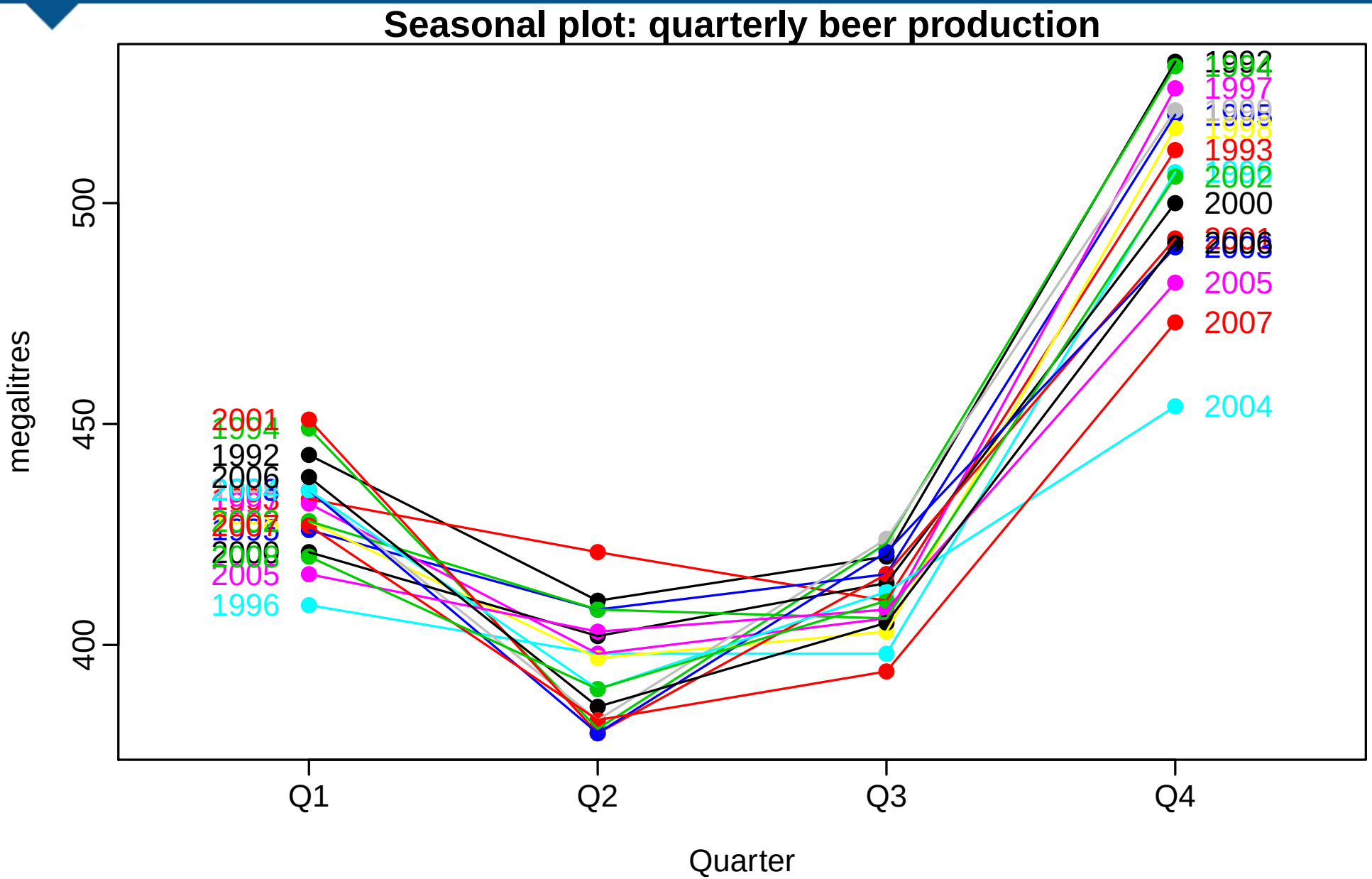

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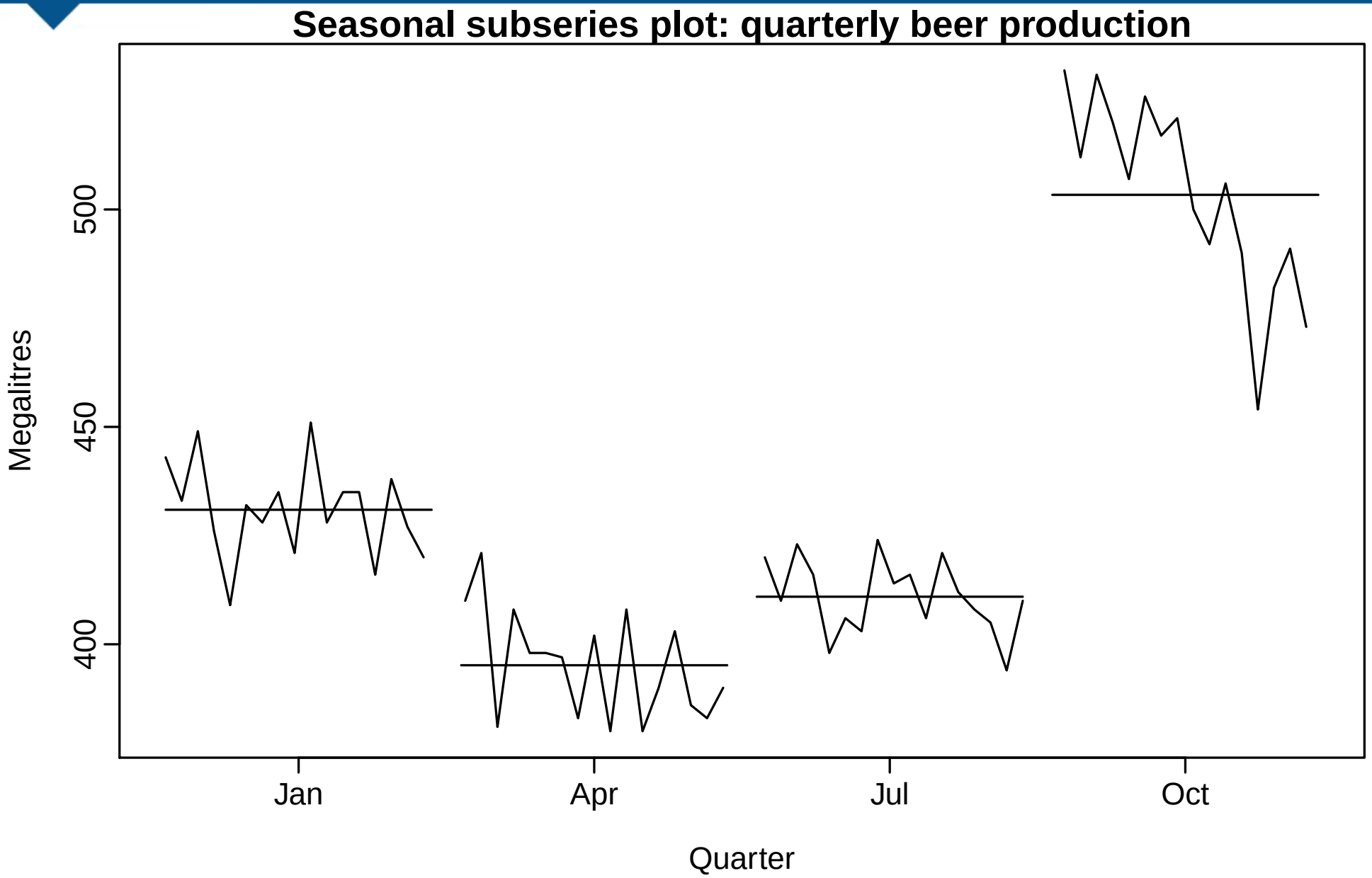

Outline

Time series patterns

Trend pattern exists when there is a long-term increase or decrease in the data.

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

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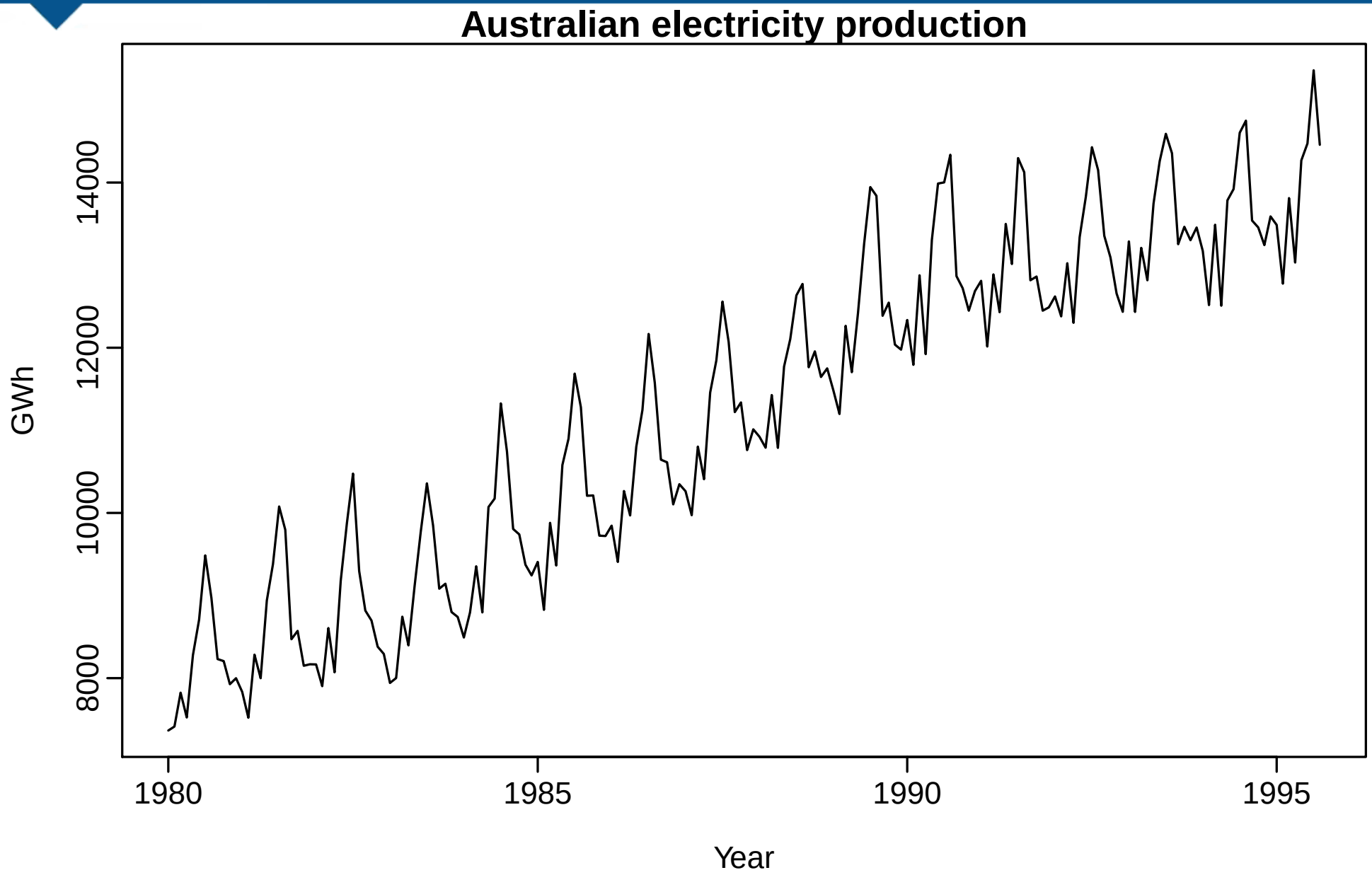

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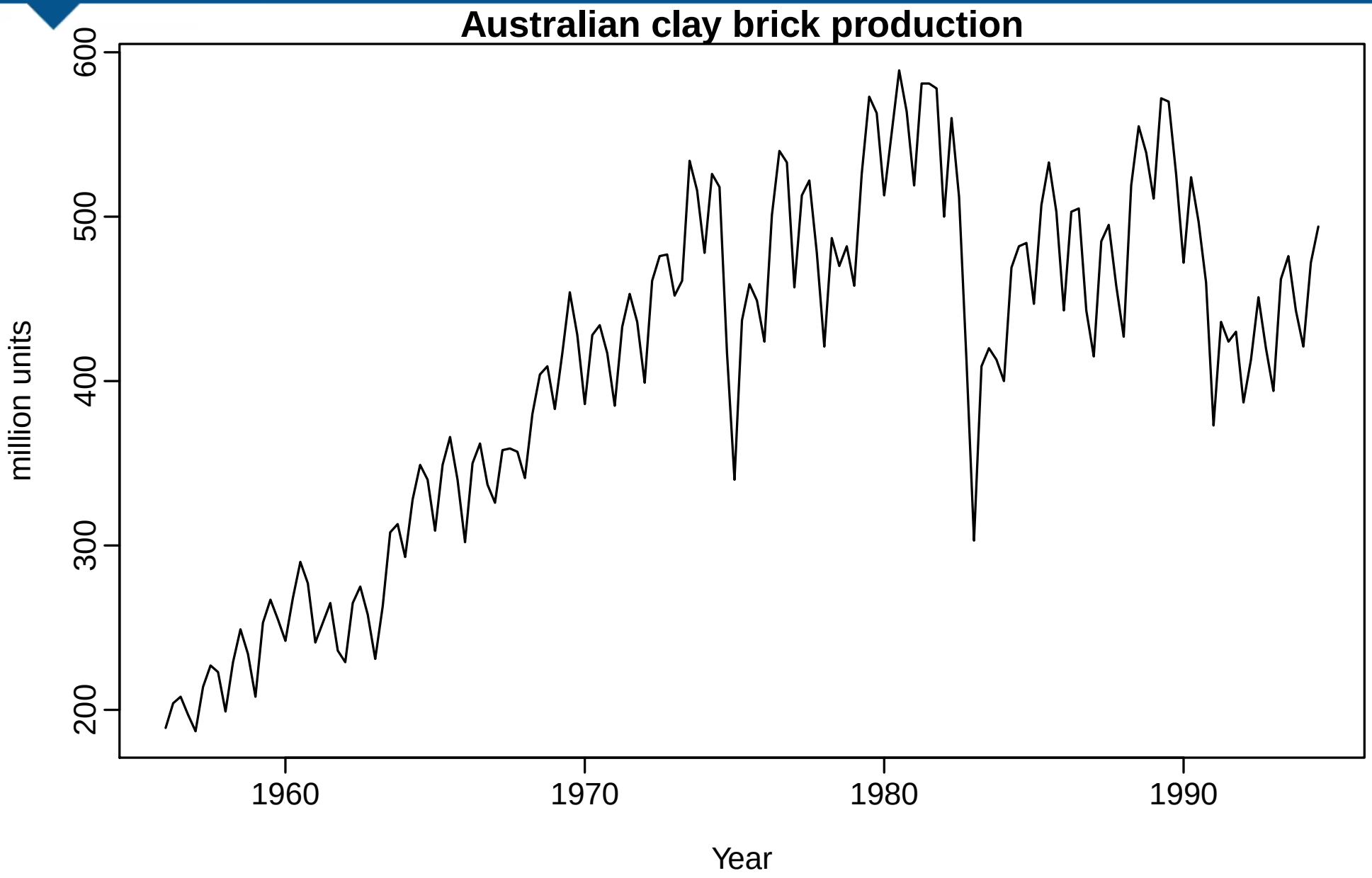

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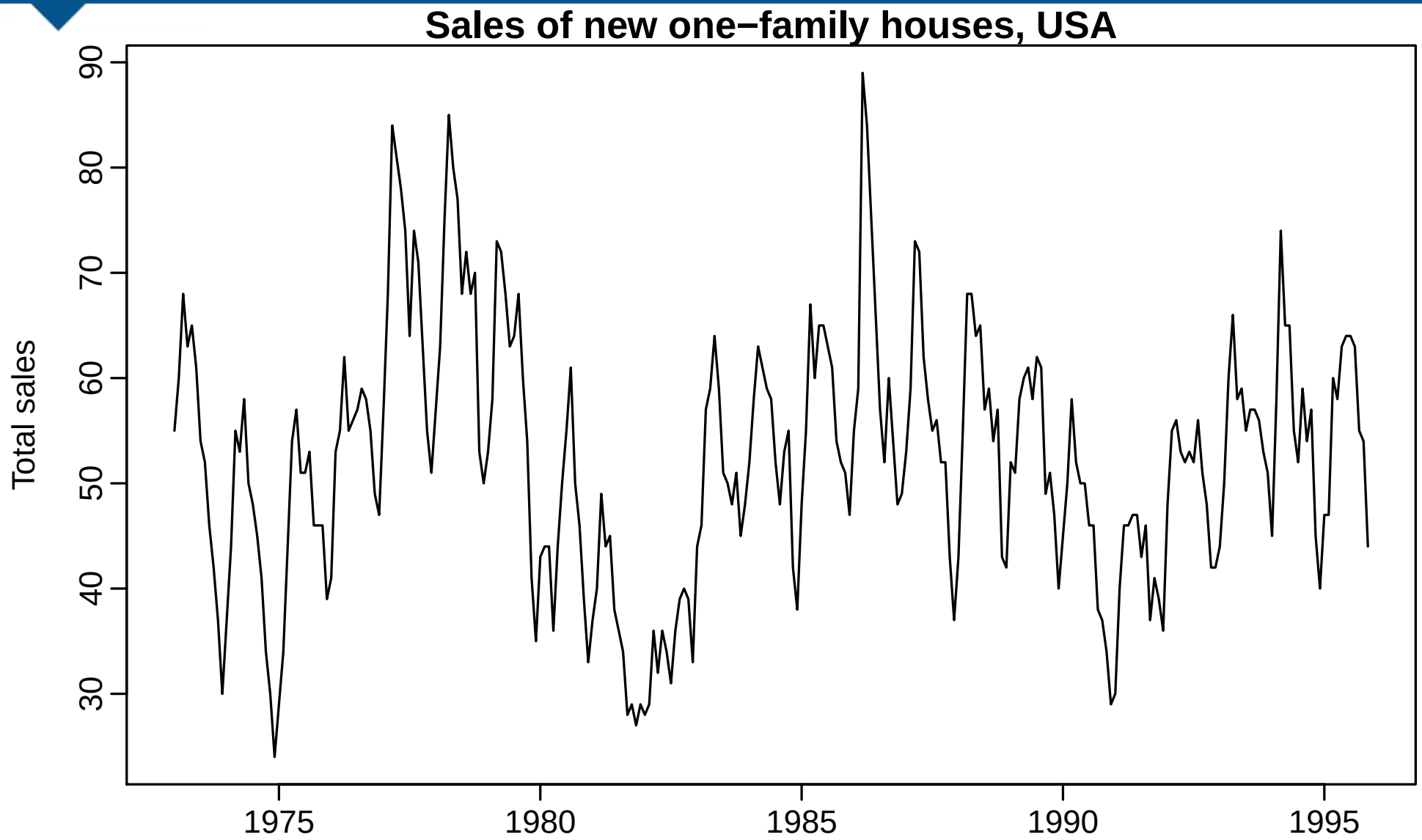

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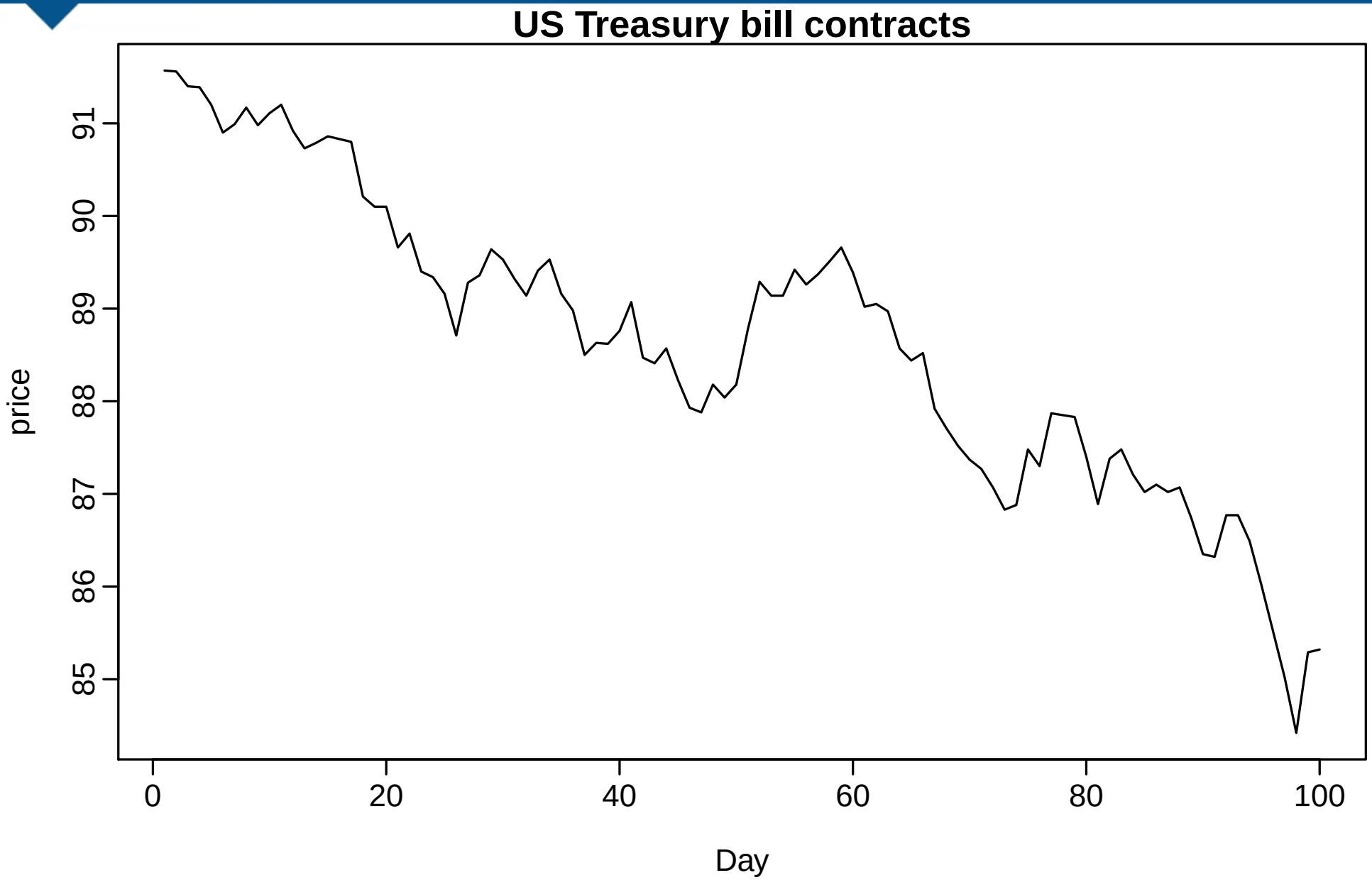

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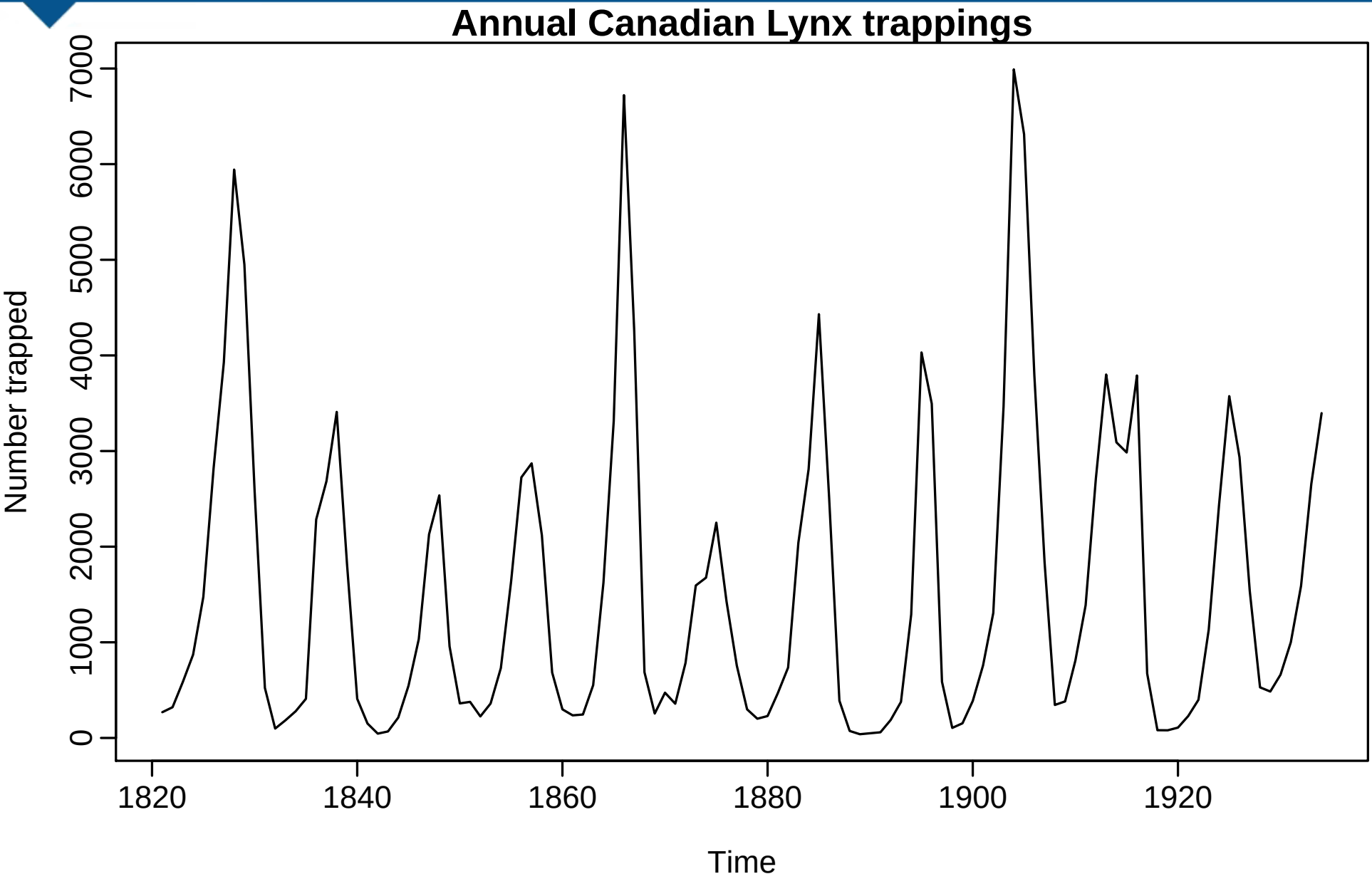

Seasonal or cyclic?

Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

The timing of peaks and troughs is predictable with seasonal data, but unpredictable in the long term with cyclic data.

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Autocorrelation

Covariance and **correlation**: measure extent of **linear relationship** between two variables (y and X).

Autocovariance and **autocorrelation**: measure linear relationship between **lagged values** of a time series y.

We measure the relationship between: y_t and y_{t-1}
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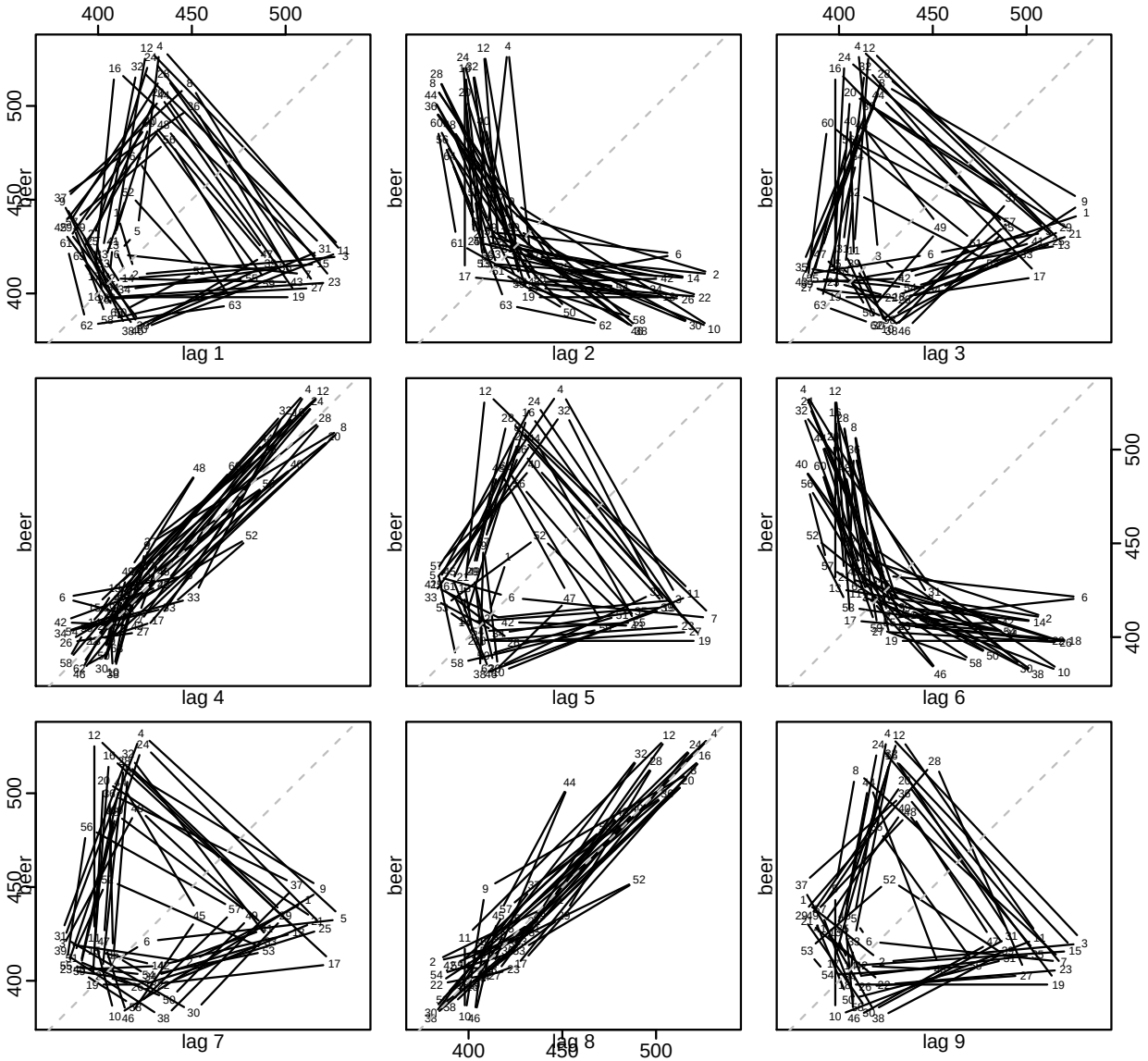
Example: Beer production

```
> lag.plot(beer,lags=9)
```



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```
> lag.plot(beer,lags=9,do.lines=FALSE)
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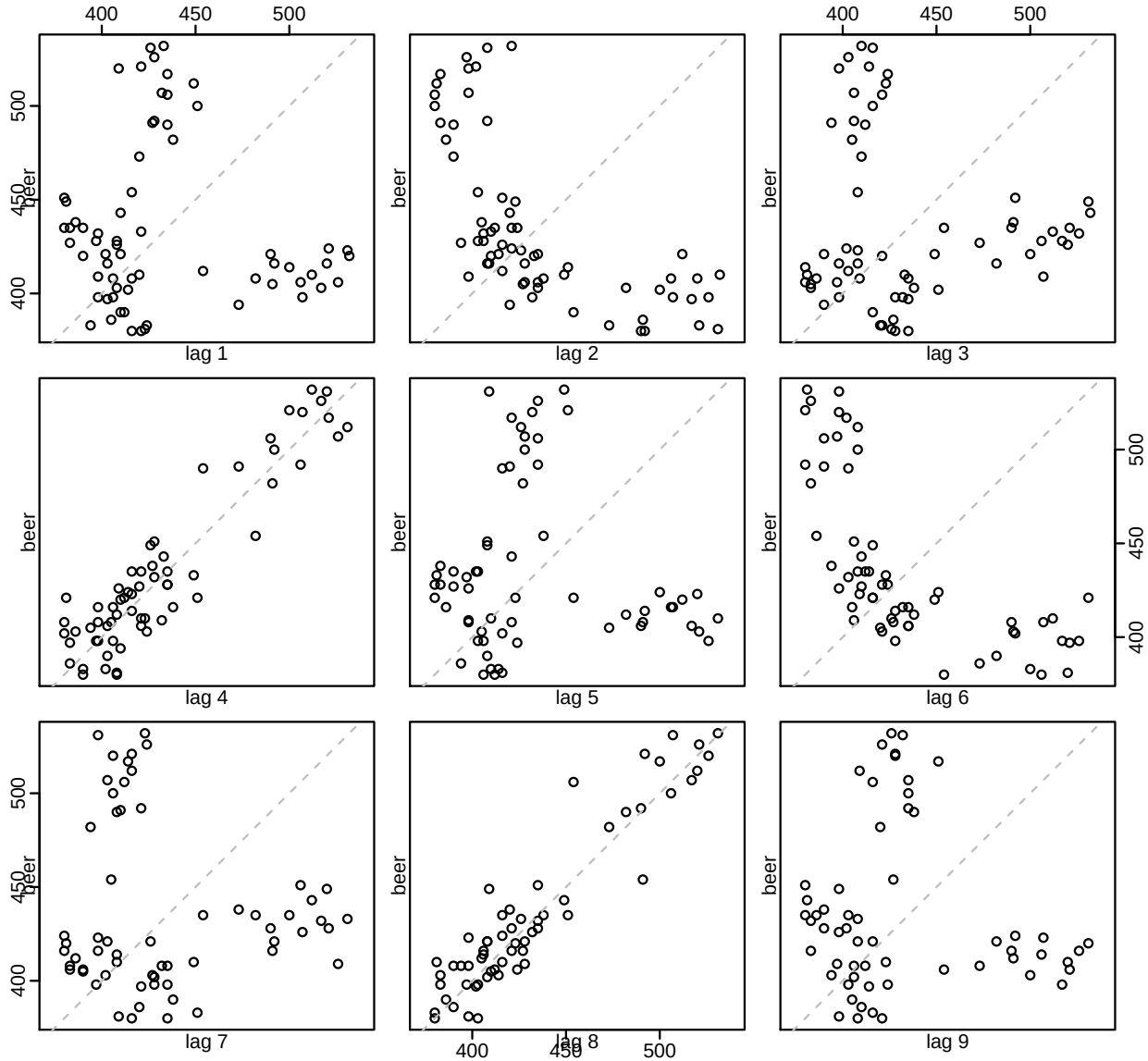

Lagged scatterplots

- Each graph shows y_t plotted against y_{t-k} for different values of k.
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Autocorrelation

We denote the sample autocovariance at lag k by c_k and the sample autocorrelation at lag k by r_k. Then define

$$c_k = \frac{1}{T}\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})$$

and $$r_k = c_k/c_0$$

- r_1 indicates how successive values of y relate to each other
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Results for first 9 lags for beer data:

r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
-0.126	-0.650	-0.094	0.863	-0.099	-0.642	-0.098	0.834	-0.116

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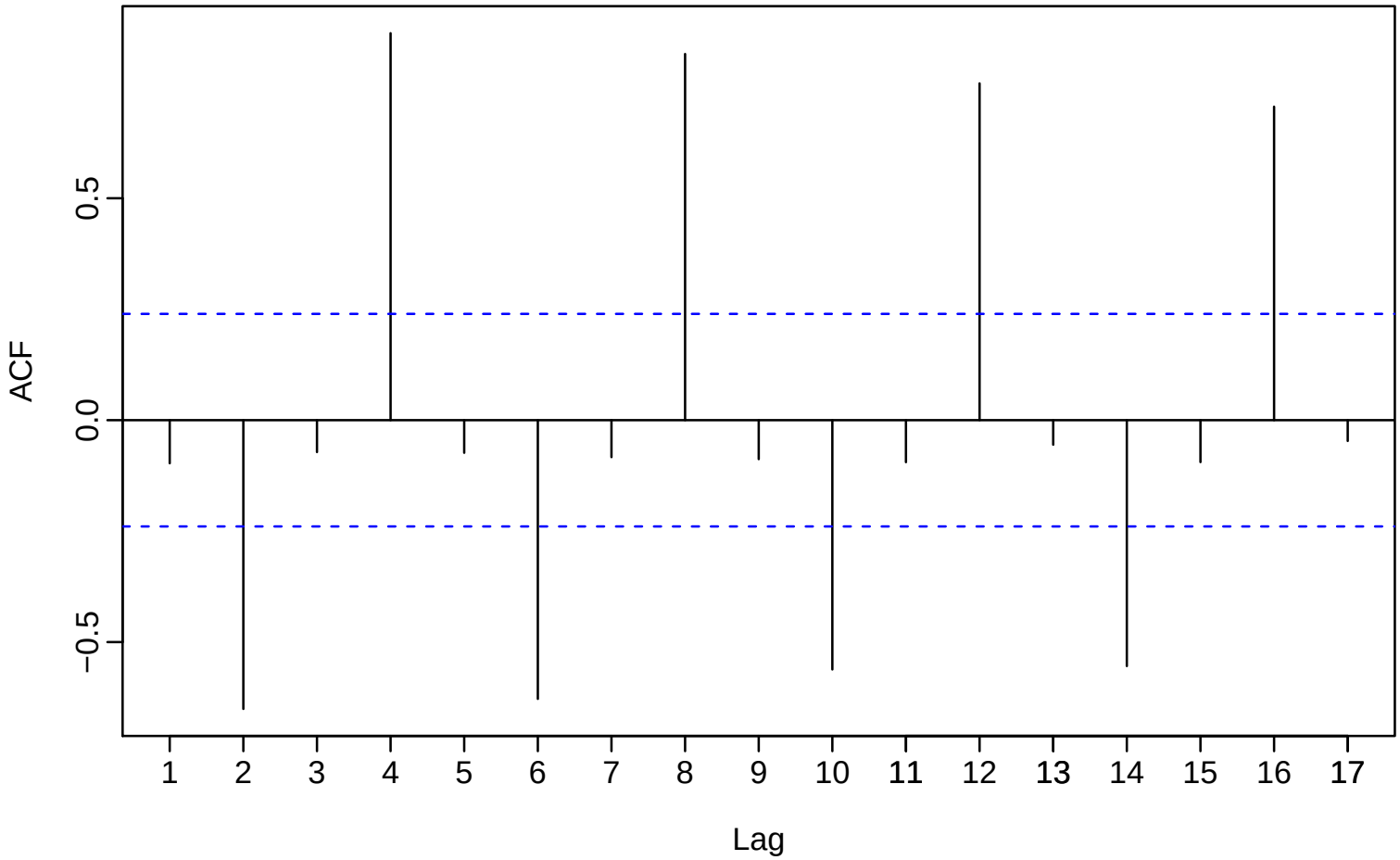

Autocorrelation

- r_4 higher than for the other lags. This is due to **the seasonal pattern in the data:** the peaks tend to be **4 quarters** apart and the troughs tend to be **2 quarters** apart.
- r_2 is more negative than for the other lags because troughs tend to be 2 quarters behind peaks.
- Together, the autocorrelations at lags 1, 2, . . . , make up the *autocorrelation* or ACF.
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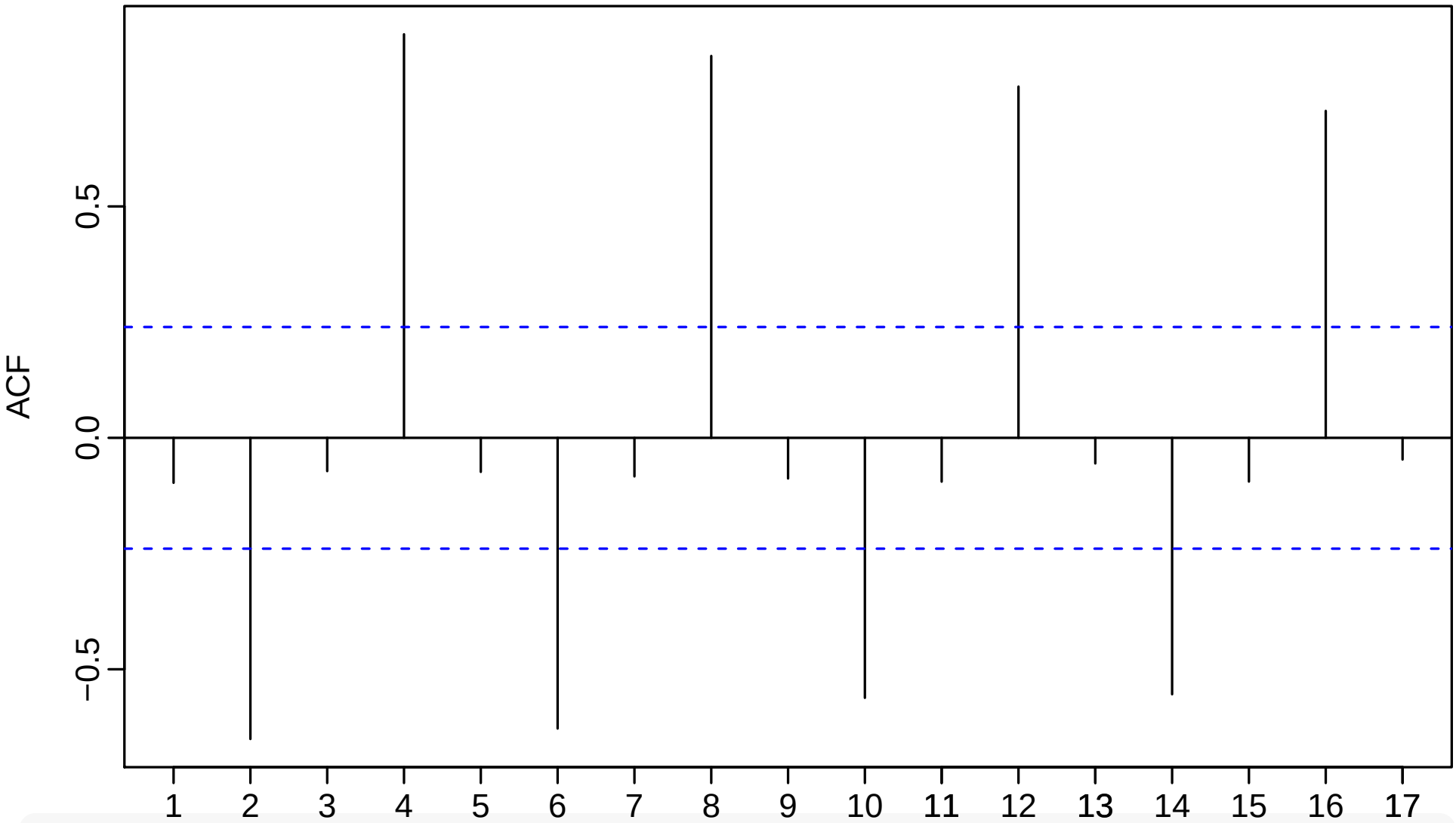


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Acf(beer)
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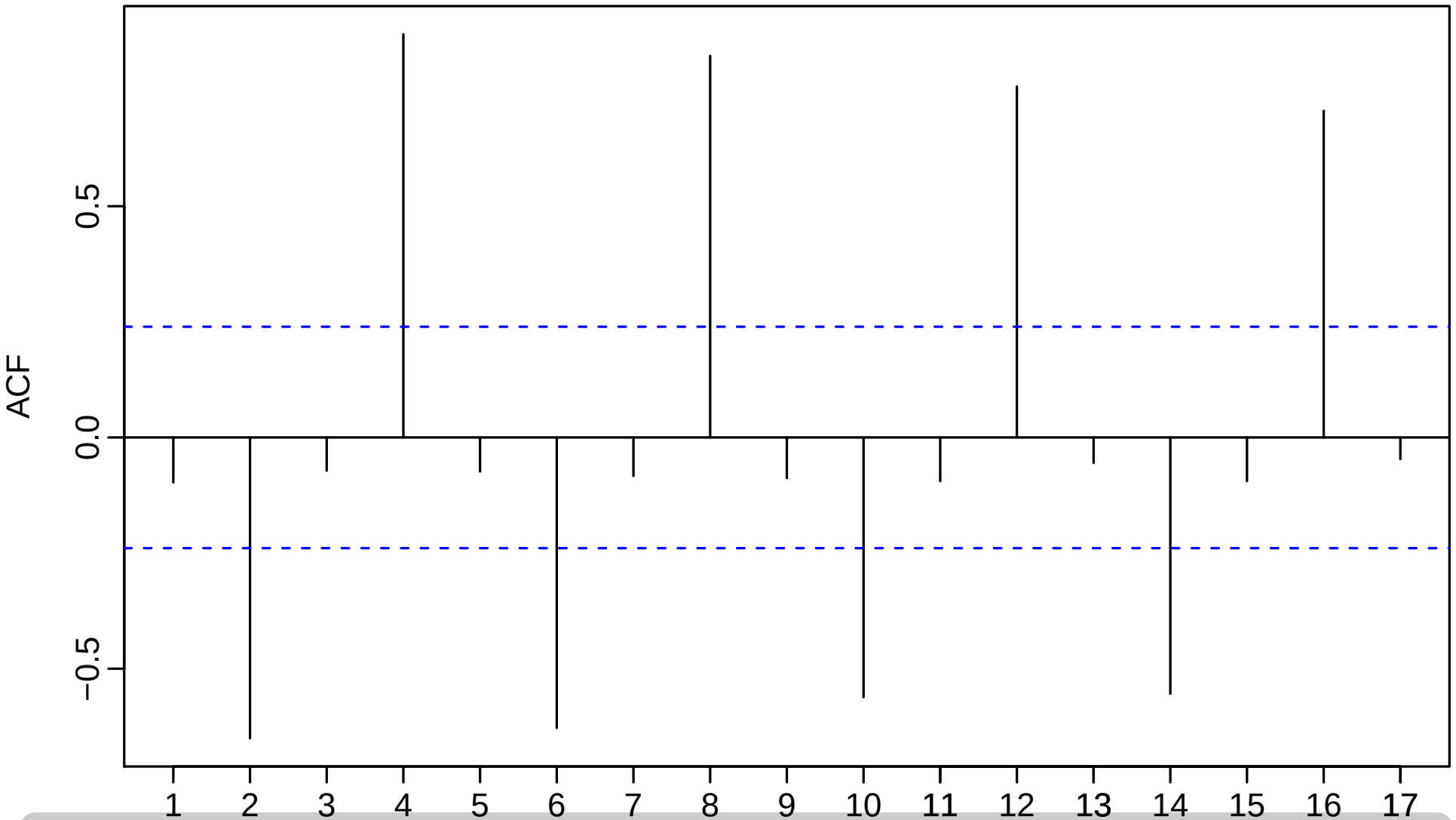


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Recognizing seasonality in a time series

If there is seasonality, the ACF at the seasonal lag (e.g., 12 for monthly data) will be **large and positive**.

- For seasonal monthly data, a large ACF value will be seen at lag 12 and possibly also at lags 24, 36, . . .
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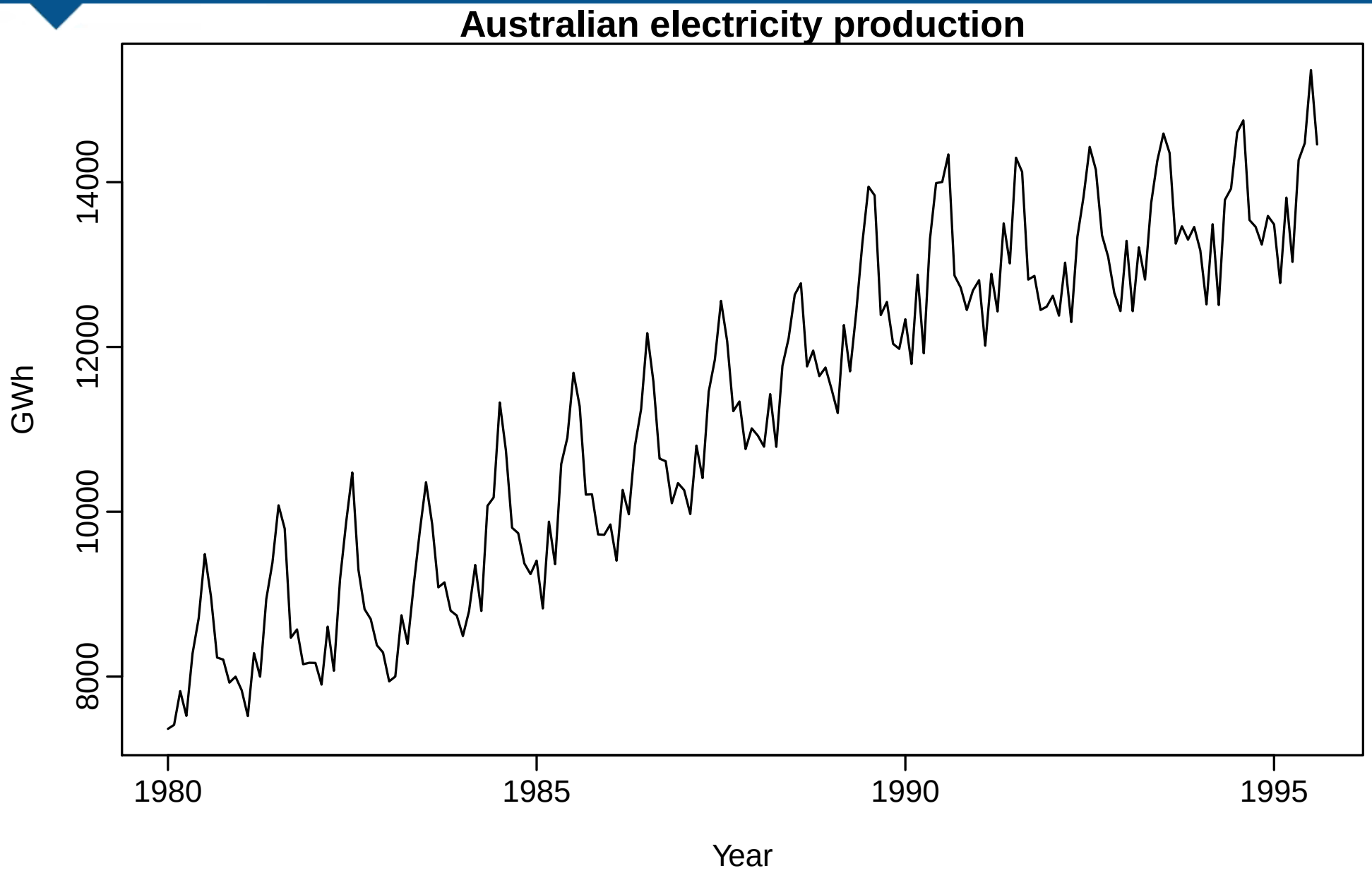

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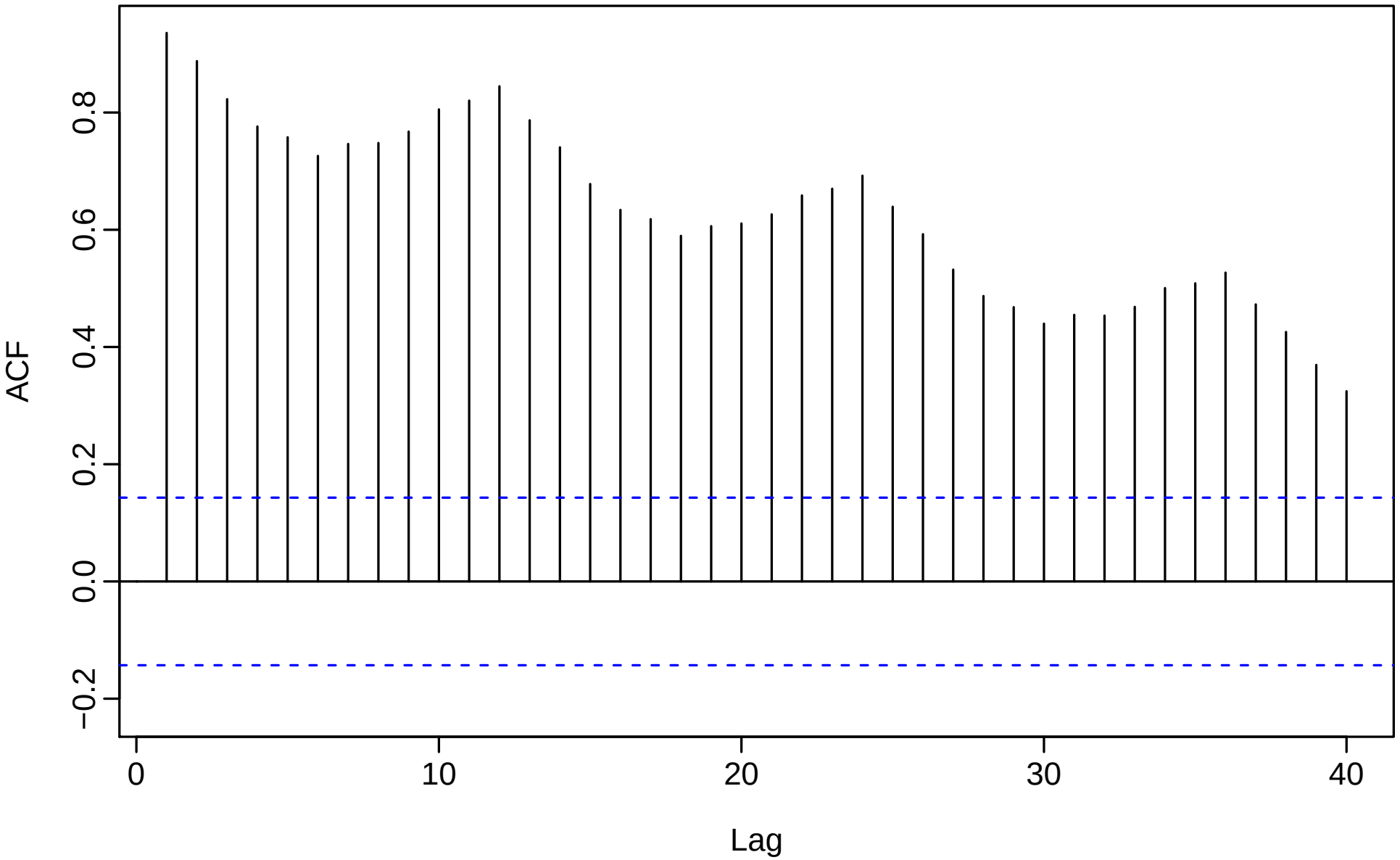

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Time plot shows clear trend and seasonality.
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- The slowly decaying ACF indicates trend.
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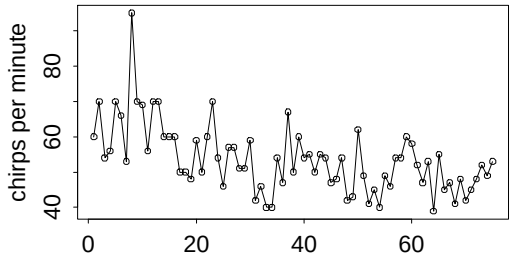
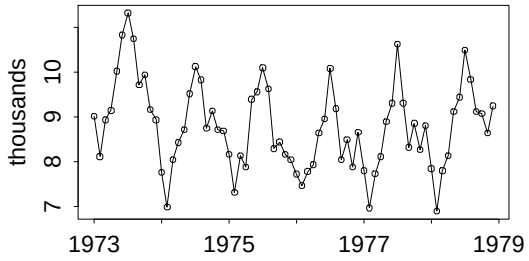
Which is which?

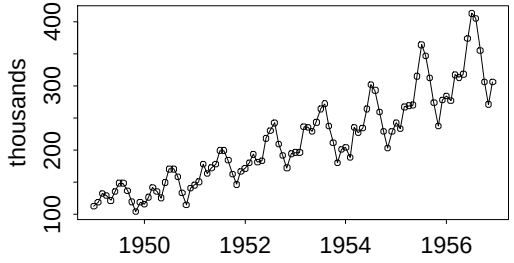
1. Daily morning temperature of a cow



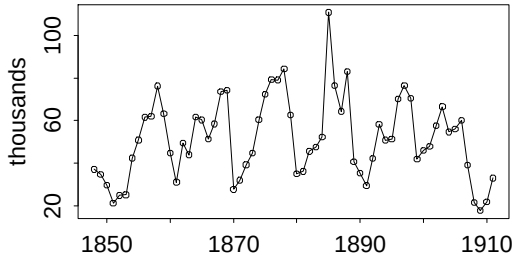
2. Accidental deaths in USA (monthly)



3. International airline passengers



4. Annual mink trappings (Canada)



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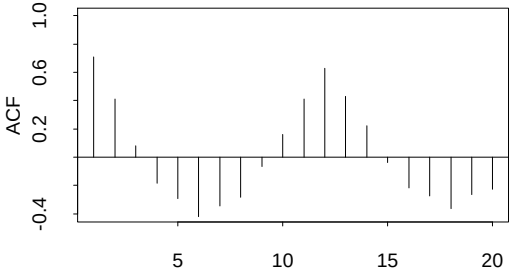

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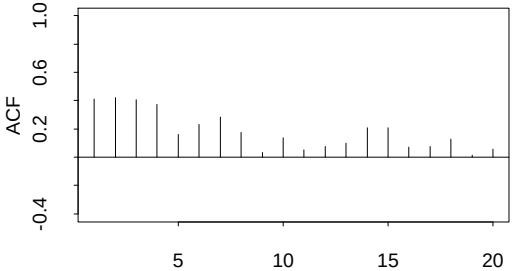
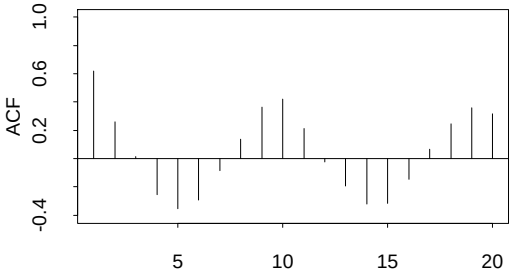
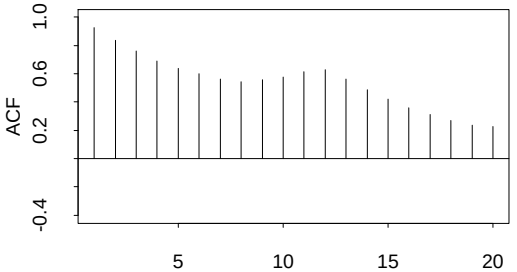


C



D



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