Rob J Hyndman

Forecasting using R

2. The forecaster's toolbox

OTexts.com/fpp/2/

Outline

Some simple forecasting methods

Average method

- Forecast of all future values is equal to mean of historical data $\{y_1, \ldots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$

Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.

Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-km}$ where $m =$ seasonal period and $k = \lfloor (h-1)/m \rfloor + 1$.

Some simple forecasting methods

Average method

- Forecast of all future values is equal to mean of historical data $\{y_1, \ldots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$

Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.

Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-km}$ where $m =$ seasonal period and $k = \lfloor (h-1)/m \rfloor + 1$.

Some simple forecasting methods

Average method

- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$

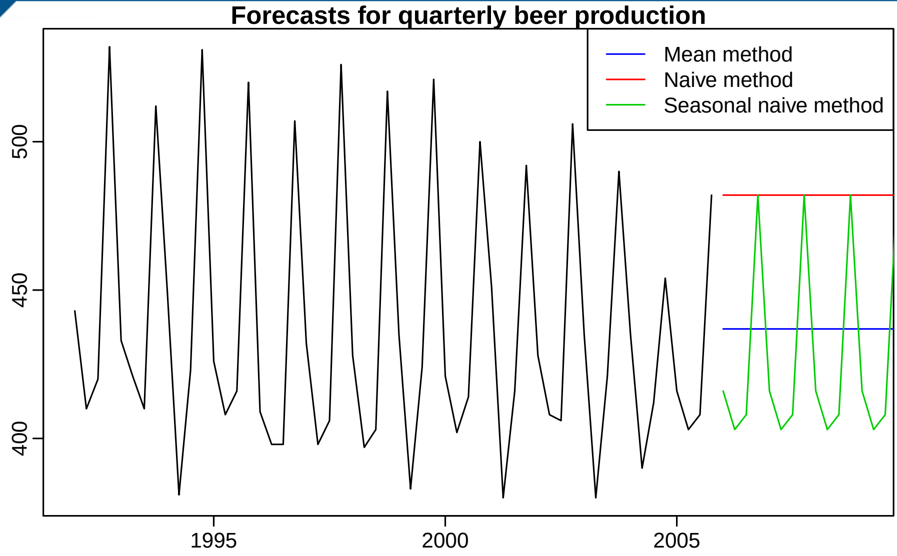
Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.

Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-km}$ where $m =$ seasonal period and $k = \lfloor (h-1)/m \rfloor + 1$.

Some simple forecasting methods



Drift method

- Forecasts equal to last value plus average change.
- Forecasts:

$$\hat{y}_{T+h|T} = y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})$$
$$= y_T + \frac{h}{T-1}(y_T - y_1).$$

- Equivalent to extrapolating a line drawn between first and last observations.

Drift method

- Forecasts equal to last value plus average change.
- Forecasts:

$$\hat{y}_{T+h|T} = y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})$$
$$= y_T + \frac{h}{T-1}(y_T - y_1).$$

- Equivalent to extrapolating a line drawn between first and last observations.

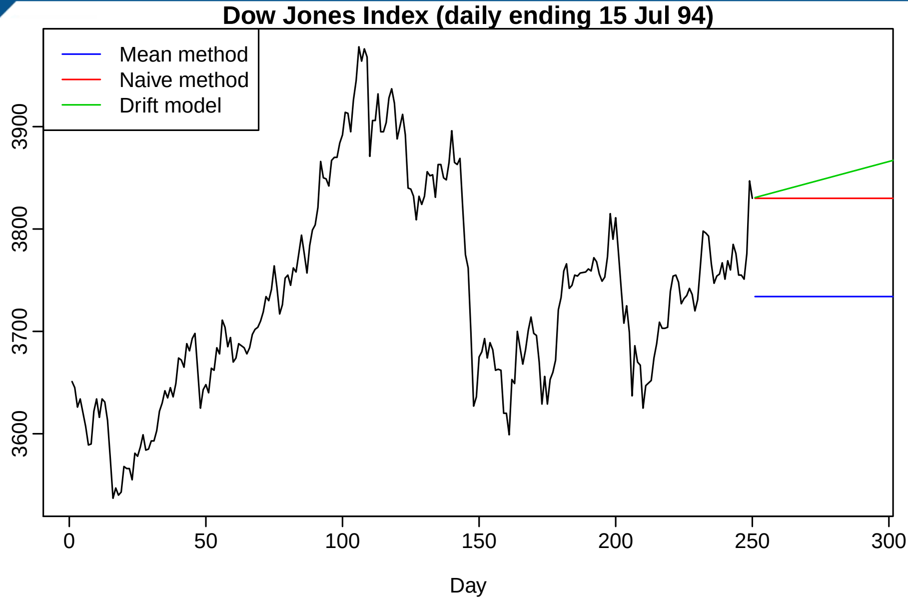
Drift method

- Forecasts equal to last value plus average change.
- Forecasts:

$$\hat{y}_{T+h|T} = y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})$$
$$= y_T + \frac{h}{T-1}(y_T - y_1).$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods



Some simple forecasting methods

- Mean: `meanf(x, h=20)`
- Naive: `naive(x, h=20)` or `rwf(x, h=20)`
- Seasonal naive: `snaive(x, h=20)`
- Drift: `rwf(x, drift=TRUE, h=20)`

Some simple forecasting methods

- Mean: `meanf(x, h=20)`
- Naive: `naive(x, h=20)` or `rwf(x, h=20)`
- Seasonal naive: `snaive(x, h=20)`
- Drift: `rwf(x, drift=TRUE, h=20)`

Some simple forecasting methods

- Mean: `meanf(x, h=20)`
- Naive: `naive(x, h=20)` or `rwf(x, h=20)`
- Seasonal naive: `snaive(x, h=20)`
- Drift: `rwf(x, drift=TRUE, h=20)`

Some simple forecasting methods

- Mean: `meanf(x, h=20)`
- Naive: `naive(x, h=20)` or `rwf(x, h=20)`
- Seasonal naive: `snaive(x, h=20)`
- Drift: `rwf(x, drift=TRUE, h=20)`

Homework 1

Any questions?

Outline

Forecasting residuals

Residuals in forecasting: difference between observed value and its forecast based on all previous observations: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

Forecasting residuals

Residuals in forecasting: difference between observed value and its forecast based on all previous observations: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

Forecasting residuals

Residuals in forecasting: difference between observed value and its forecast based on all previous observations: $e_t = y_t - \hat{y}_{t|t-1}$.

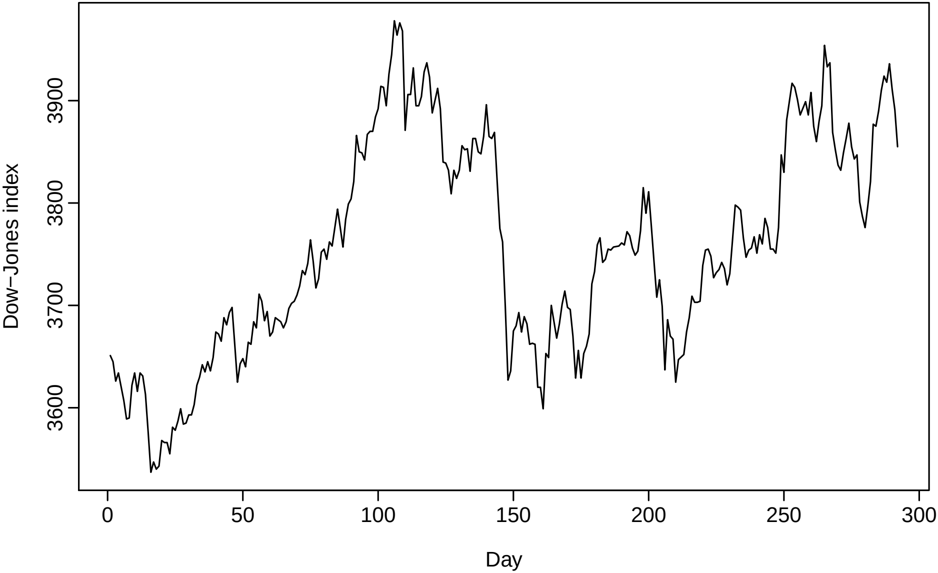
Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

Forecasting Dow-Jones index



Forecasting Dow-Jones index

Naïve forecast:

$$\hat{y}_{t|t-1} = y_{t-1}$$

$$e_t = y_t - y_{t-1}$$

Note: e_t are one-step-forecast residuals

Forecasting Dow-Jones index

Naïve forecast:

$$\hat{y}_{t|t-1} = y_{t-1}$$

$$e_t = y_t - y_{t-1}$$

Note: e_t are one-step-forecast residuals

Forecasting Dow-Jones index

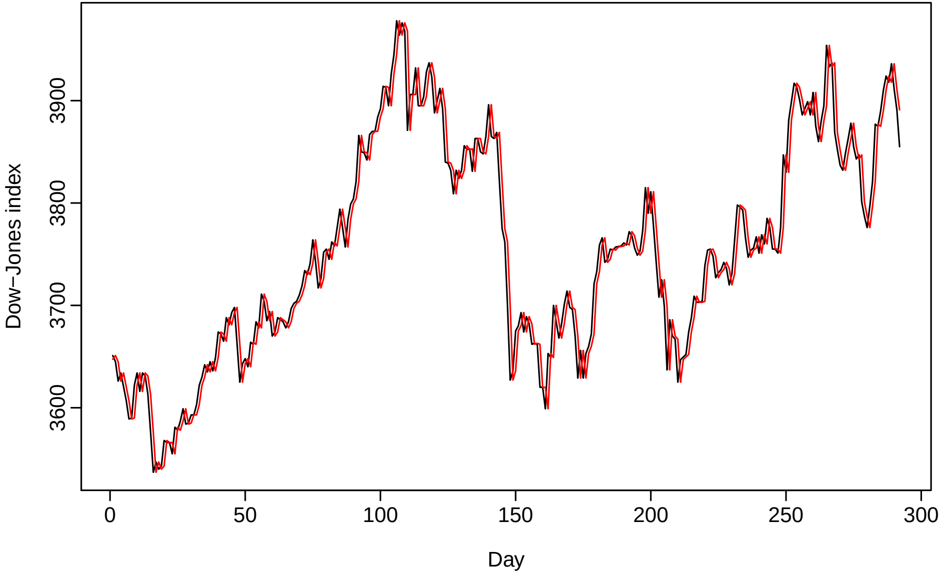
Naïve forecast:

$$\hat{y}_{t|t-1} = y_{t-1}$$

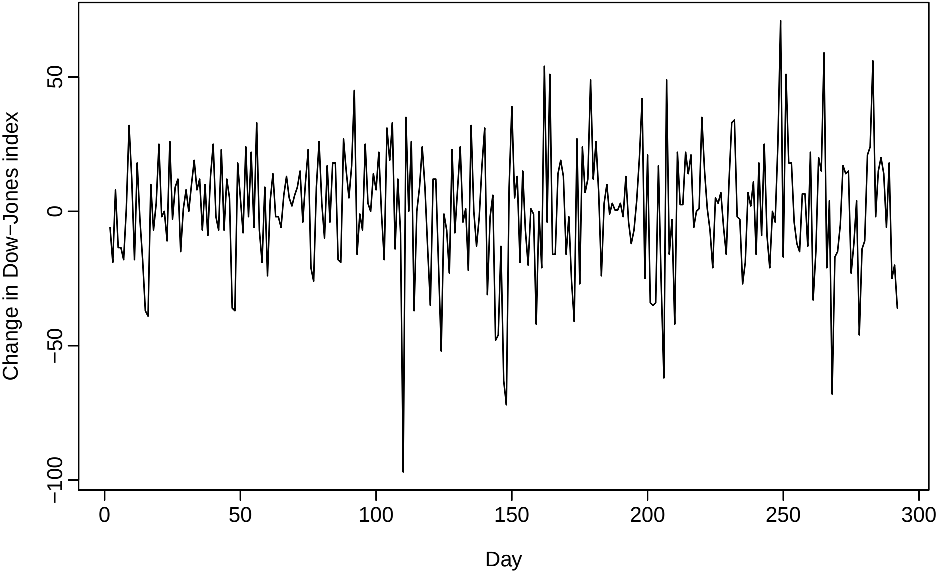
$$e_t = y_t - y_{t-1}$$

Note: e_t are one-step-forecast residuals

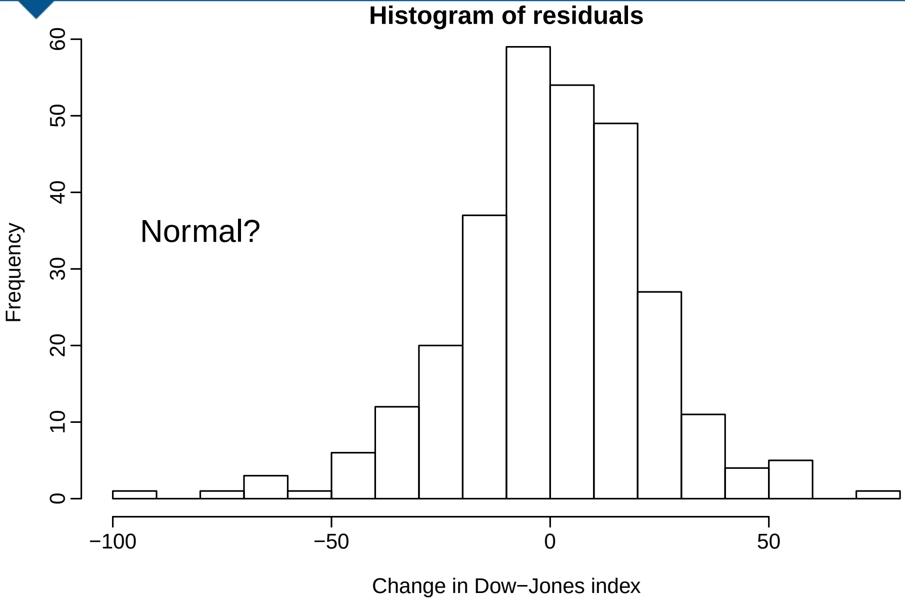
Forecasting Dow-Jones index



Forecasting Dow-Jones index



Forecasting Dow-Jones index



Outline

Measures of forecast accuracy

Let y_t denote the tth observation and $\hat{y}_{t|t-1}$ denote its forecast based on all previous data, where $t = 1, \dots, T$. Then the following measures are useful.

$$\text{MAE} = T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|$$

$$\text{MSE} = T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2 \qquad \text{RMSE} = \sqrt{T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2}$$

$$\text{MAPE} = 100T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/|y_t|$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Let y_t denote the tth observation and $\hat{y}_{t|t-1}$ denote its forecast based on all previous data, where $t = 1, \dots, T$. Then the following measures are useful.

$$\text{MAE} = T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|$$

$$\text{MSE} = T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2 \qquad \text{RMSE} = \sqrt{T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2}$$

$$\text{MAPE} = 100T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/|y_t|$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Let y_t denote the tth observation and $\hat{y}_{t|t-1}$ denote its forecast based on all previous data, where $t = 1, \dots, T$. Then the following measures are useful.

$$\text{MAE} = T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|$$

$$\text{MSE} = T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2 \qquad \text{RMSE} = \sqrt{T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2}$$

$$\text{MAPE} = 100T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/|y_t|$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}| / Q$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/Q$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006)

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/Q$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

For non-seasonal time series,

$$Q = (T-1)^{-1} \sum_{t=2}^{T} |y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naive method.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/Q$$

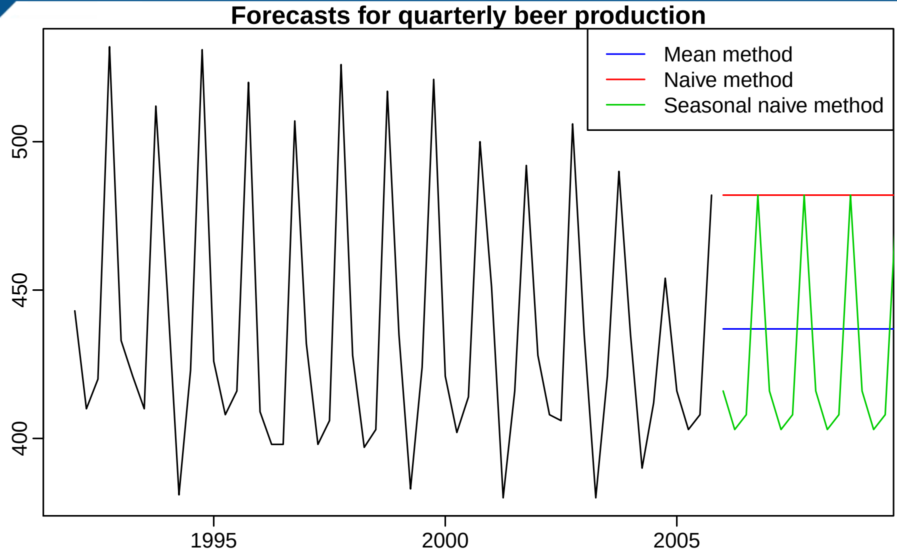
where Q is a stable measure of the scale of the time series $\{y_t\}$.

For seasonal time series,

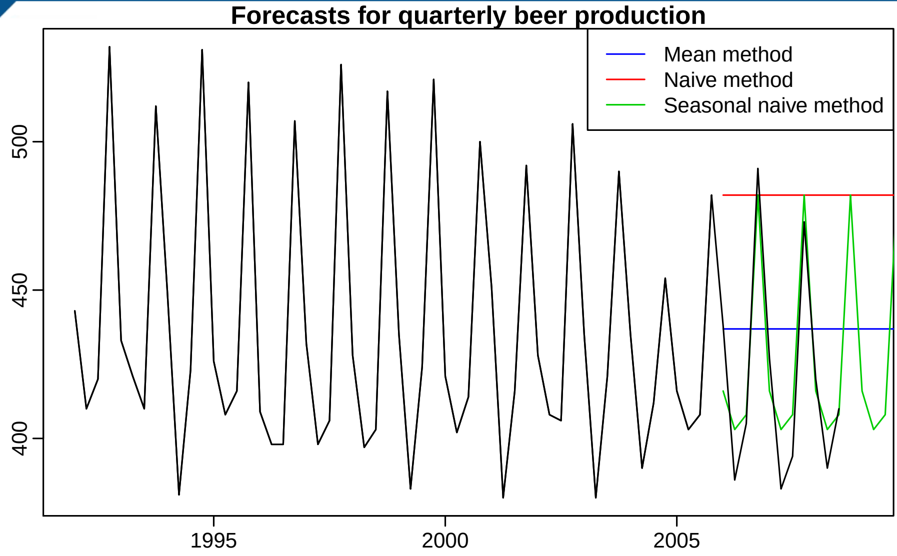
$$Q = (T - m)^{-1} \sum_{t=m+1}^{T} |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naive method.

Measures of forecast accuracy



Measures of forecast accuracy



Measures of forecast accuracy

Mean method

RMSE	MAE	MAPE	MASE
38.0145	33.7776	8.1700	2.2990

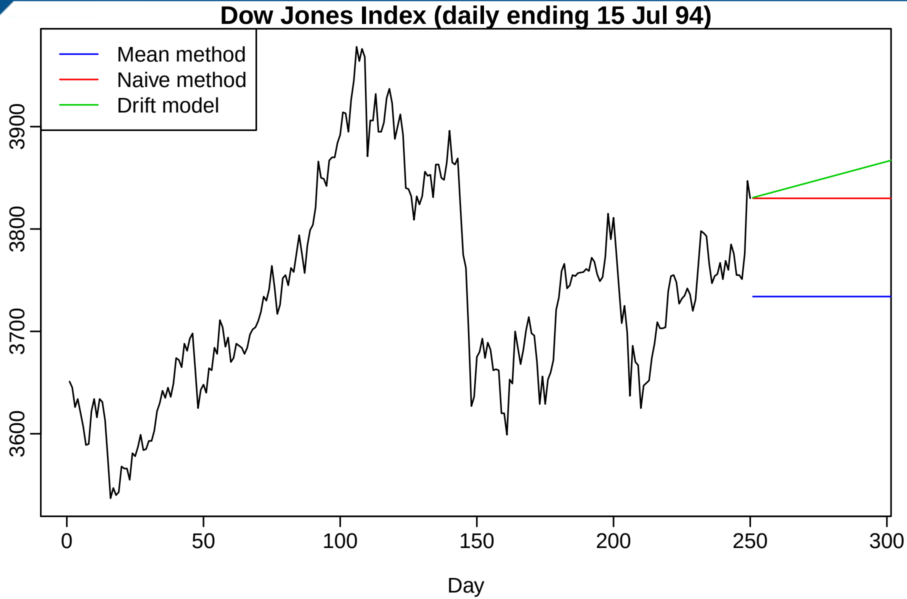
Naïve method

RMSE	MAE	MAPE	MASE
70.9065	63.9091	15.8765	4.3498

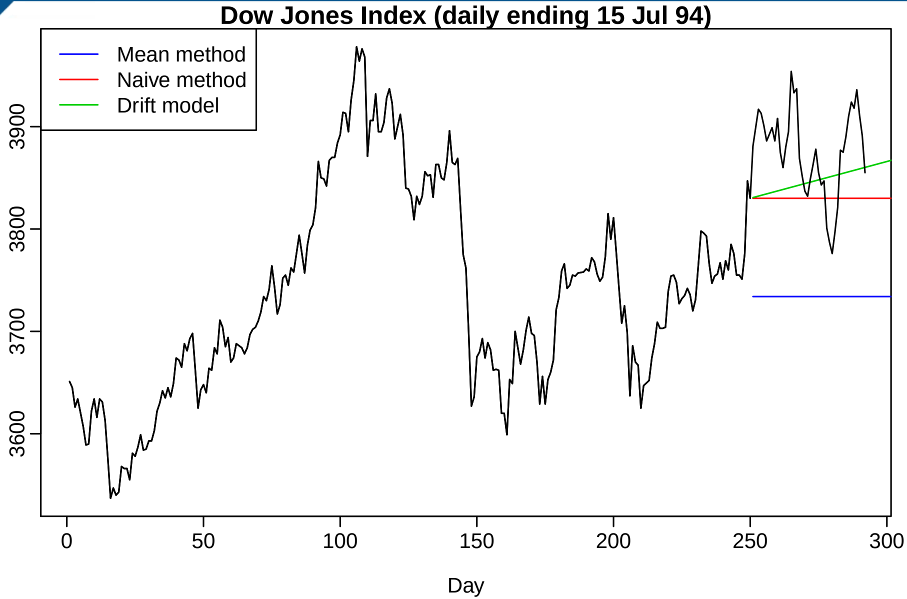
Seasonal naïve method

RMSE	MAE	MAPE	MASE
12.9685	11.2727	2.7298	0.7673

Measures of forecast accuracy



Measures of forecast accuracy



Measures of forecast accuracy

Mean method

RMSE	MAE	MAPE	MASE
148.2357	142.4185	3.6630	8.6981

Naïve method

RMSE	MAE	MAPE	MASE
62.0285	54.4405	1.3979	3.3249

Drift model

RMSE	MAE	MAPE	MASE
53.6977	45.7274	1.1758	2.7928

Training and test sets

Available data

Training set (e.g., 80%)	Test set (e.g., 20%)

- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Training and test sets

Available data

Training set (e.g., 80%)	Test set (e.g., 20%)

- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Training and test sets

```r
beer3 <- window(ausbeer,start=1992,end=2005.99)
beer4 <- window(ausbeer,start=2006)

fit1 <- meanf(beer3,h=20)
fit2 <- rwf(beer3,h=20)

accuracy(fit1,beer4)
accuracy(fit2,beer4)
```

In-sample accuracy (one-step forecasts)

```r
accuracy(fit1)
accuracy(fit2)
```

Training and test sets

```
beer3 <- window(ausbeer,start=1992,end=2005.99)
beer4 <- window(ausbeer,start=2006)

fit1 <- meanf(beer3,h=20)
fit2 <- rwf(beer3,h=20)

accuracy(fit1,beer4)
accuracy(fit2,beer4)
```

In-sample accuracy (one-step forecasts)

```
accuracy(fit1)
accuracy(fit2)
```

Beware of over-fitting

- A model which fits the data well does not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters. (Compare R^2)
- Over-fitting a model to data is as bad as failing to identify the systematic pattern in the data.
- Problems can be overcome by measuring true *out-of-sample* forecast accuracy. That is, total data divided into “training” set and “test” set. Training set used to estimate parameters. Forecasts are made for test set.
- Accuracy measures computed for errors in test set only.

Beware of over-fitting

- A model which fits the data well does not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters. (Compare R^2)
- Over-fitting a model to data is as bad as failing to identify the systematic pattern in the data.
- Problems can be overcome by measuring true *out-of-sample* forecast accuracy. That is, total data divided into "training" set and "test" set. Training set used to estimate parameters. Forecasts are made for test set.
- Accuracy measures computed for errors in test set only.

Beware of over-fitting

- A model which fits the data well does not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters. (Compare R^2)
- Over-fitting a model to data is as bad as failing to identify the systematic pattern in the data.
- Problems can be overcome by measuring true *out-of-sample* forecast accuracy. That is, total data divided into "training" set and "test" set. Training set used to estimate parameters. Forecasts are made for test set.
- Accuracy measures computed for errors in test set only.

Beware of over-fitting

- A model which fits the data well does not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters. (Compare R^2)
- Over-fitting a model to data is as bad as failing to identify the systematic pattern in the data.
- Problems can be overcome by measuring true *out-of-sample* forecast accuracy. That is, total data divided into “training” set and “test” set. Training set used to estimate parameters. Forecasts are made for test set.
- Accuracy measures computed for errors in test set only.

Beware of over-fitting

- A model which fits the data well does not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters. (Compare R^2)
- Over-fitting a model to data is as bad as failing to identify the systematic pattern in the data.
- Problems can be overcome by measuring true *out-of-sample* forecast accuracy. That is, total data divided into “training” set and “test” set. Training set used to estimate parameters. Forecasts are made for test set.
- Accuracy measures computed for errors in test set only.

Poll: true or false?

1. Good forecast methods should have normally distributed residuals.
2. A model with small residuals will give good forecasts.
3. The best measure of forecast accuracy is MAPE.
4. If your model doesn’t forecast well, you should make it more complicated.
5. Always choose the model with the best forecast accuracy as measured on the test set.