Population forecasting and the importance of being uncertain

Rob J Hyndman
2007 Knibbs Lecture

Sir George Handley Knibbs
(1858–1929)

- President of the Institute of Surveyors, 1892–1901
- President of the British Astronomical Society (NSW), 1897–1898
- President of the Royal Society of NSW, 1898–1899
- Lecturer in Geodesy, Astronomy and Hydraulics, Uni of Sydney, 1889–1905
- Director of Technical Education, NSW, 1905
- Professor of Physics, Uni of Sydney, 1905
- Commonwealth Statistician, 1906–1921
- Director of Institute of Science and Industry, 1921–1926
Outline

1. The dodgy history of forecasting
2. Projections and what-if scenarios
3. Exponential smoothing
4. Forecasting Australia’s population
5. Conclusions
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What is it?
What is it?

Clay model of sheep’s liver

Used by Babylonian forecasters approximately 600 B.C.

Now in British Museum.
Delphic oracle
Delphic oracle
Delphic oracle
Vagrant forecasters

The British Vagrancy Act (1736) made it an offence to defraud by charging money for predictions.
Vagrant forecasters

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**Punishment:** a fine or three months’ imprisonment with hard labour.
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Standard business practice today

Graphic Forecaster

Create forecasts visually with a "drag and drop" graphic forecaster. The Graphic Forecaster is a simple and powerful tool to streamline the forecasting process. You can change your sales and expenses estimates by simply clicking your mouse button to move the line on your forecast chart or apply a specific growth rate to the whole year. Build forecasts using visual common sense.
A huge advance over spreadsheet-based systems, Budget Maestro is a complete solution for budgeting, forecasting, what-if scenario planning, reporting and analysis. Budget Maestro takes the pain out of the budgeting process (no tedious data entry and formula verification) while providing you a tool to more accurately analyze and measure business performance and profitability. Budget Maestro’s capabilities include:

**Budgeting and Forecasting:** Budget Maestro utilizes database technology for real-time data collection and reporting. A common interface for all users fosters collaboration and increases the accuracy of data entry. There are no formulas or macros to create, no tedious re-keying of data and no mystery links to chase down and fix. Budget Maestro’s built-in “financial intelligence and business rules” builds the formulas for you ensuring 100% accuracy.
Standard business practice today

- “What-if scenarios” based on assumed and fixed future conditions.
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Is this any better than a sheep’s liver or hallucinogens?
The Australian Bureau of Statistics provide population “projections”.

“The projections are not intended as predictions or forecasts, but are illustrations of growth and change in the population that would occur if assumptions made about future demographic trends were to prevail over the projection period. While the assumptions are formulated on the basis of an assessment of past demographic trends, both in Australia and overseas, there is no certainty that any of the assumptions will be realised. In addition, no assessment has been made of changes in non-demographic conditions.”

ABS 3222.0 - Population Projections, Australia, 2004 to 2101
ABS population projections

The ABS provides three projection scenarios labelled “High”, “Medium” and “Low”.

- Based on assumed mortality, fertility and migration rates

No objectivity.
No dynamic changes in rates allowed
Arbitrary adjustments to some rates
No probabilistic basis.
Not prediction intervals.
Most users use the “Medium” projection, but it is unrelated to the mean, median or mode of the future distribution.
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What do these projections mean?
What do these projections mean?
Advantages of stochastic models

Based on empirical data
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- Based on empirical data
- Computable
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- Based on empirical data
- Computable
- Replicable
Advantages of stochastic models

- Based on empirical data
- Computable
- Replicable
- Testable
Advantages of stochastic models

- Based on empirical data
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- **Objective measure of uncertainty**
Advantages of stochastic models

- Based on empirical data
- Computable
- Replicable
- Testable
- Objective measure of uncertainty
- Able to compute prediction intervals
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1. The dodgy history of forecasting
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Exponential smoothing is extremely popular, simple to implement, and performs well in forecasting competitions.
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“Unfortunately, exponential smoothing methods do not allow easy calculation of prediction intervals.”

Since 2002...

- A general class of state space models proposed underlying all the common exponential smoothing methods.
- Analytical results for prediction intervals.
- Likelihood calculation for estimation.
- AIC for model selection.
- An algorithm for automatic forecasting using the new class of models.
- New results on the admissible parameter space.
Exponential smoothing

- enables easy calculation of the likelihood
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- provides facilities to compute (analytical) prediction intervals
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- two possible state space models for each method (additive error and multiplicative error). Equivalent point forecasts, different prediction intervals.
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Implementation

This methodology is available as
- An R package (forecast) on CRAN
- An Excel add-in (PhiCast)
New book due out in 2008

Forecasting with exponential smoothing: the state space approach

Rob J. Hyndman
Anne B. Koehler
J. Keith Ord
Ralph D. Snyder

Published by Springer
Forecasting and the importance of being uncertain

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Annual age-specific population

Australia: male population (1921–2004)
Forecasting and the importance of being uncertain

Annual age-specific population

Australia: female population (1921–2004)
Stochastic population forecasts

Key ideas

- Population is a function of mortality, fertility and net migration.
Stochastic population forecasts

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- Population is a function of **mortality**, **fertility** and **net migration**.
- Build an age-sex-specific **stochastic model** for each of mortality, fertility & net migration.
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- Use the models to **simulate future sample paths** of all components.
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- Population is a function of mortality, fertility and net migration.
- Build an age-sex-specific stochastic model for each of mortality, fertility & net migration.
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- Compute future births, deaths, net migrants and populations from simulated rates.
Stochastic population forecasts

Key ideas

- Population is a function of **mortality**, **fertility** and **net migration**.
- Build an age-sex-specific **stochastic model** for each of mortality, fertility & net migration.
- Use the models to **simulate future sample paths** of all components.
- Compute future births, deaths, net migrants and populations from simulated rates.
- Combine the results to get **age-specific stochastic population forecasts**.
Demographic growth-balance equation

\[ P_{t+1}(x+1) = P_t(x) - D_t(x, x+1) + G_t(x, x+1) \]

\[ P_{t+1}(0) = B_t - D_t(B, 0) + G_t(B, 0) \]

\( x = 0, 1, 2, \ldots \)
Demographic growth-balance equation

\[ P_{t+1}(x + 1) = P_t(x) - D_t(x, x + 1) + G_t(x, x + 1) \]
\[ P_{t+1}(0) = B_t - D_t(B, 0) + G_t(B, 0) \]
\[ x = 0, 1, 2, \ldots \]

\[ P_t(x) = \text{population of age } x \text{ at 1 January, year } t \]
\[ B_t = \text{births in calendar year } t \]
\[ D_t(x, x + 1) = \text{deaths in calendar year } t \text{ of persons aged } x \text{ at the beginning of year } t \]
\[ D_t(B, 0) = \text{deaths in calendar year } t \text{ of persons born in year } t \]
\[ G_t(x, x + 1) = \text{net migrants in calendar year } t \text{ of persons aged } x \text{ at the beginning of year } t \]
\[ G_t(B, 0) = \text{net migrants of infants born in calendar year } t \]
The available data

The following data are available:

\[ P_t(x) = \text{population of age } x \text{ at 1 January, year } t \]
\[ E_t(x) = \text{population of age } x \text{ at 30 June, year } t \]
\[ B_t(x) = \text{births in calendar year } t \text{ to females of age } x \]
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From these, we can estimate:

\[ m_t(x) = D_t(x)/E_t(x) = \text{central death rate in calendar year } t; \]
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From these, we can estimate:

- \[ m_t(x) = D_t(x)/E_t(x) = \text{central death rate in calendar year } t; \]
- \[ f_t(x) = B_t(x)/E_t^F(x) = \text{fertility rate for females of age } x \text{ in calendar year } t. \]
The available data

The following data are available:

- $P_t(x) =$ **population** of age $x$ at 1 January, year $t$
- $E_t(x) =$ **population** of age $x$ at 30 June, year $t$
- $B_t(x) =$ **births** in calendar year $t$ to females of age $x$
- $D_t(x) =$ **deaths** in calendar year $t$ of persons of age $x$

From these, we can estimate:

- $m_t(x) = D_t(x)/E_t(x) =$ central death rate in calendar year $t$;
- $f_t(x) = B_t(x)/E_t^F(x) =$ fertility rate for females of age $x$ in calendar year $t$.
- $D_t(x, x + 1)$ and $D_t(B, 0)$. 

Mortality rates

Australia: male mortality (1921)
Mortality rates

Australia: male death rates (1921–2003)
Mortality rates

Australia: female death rates (1921–2003)

Log death rate

Age
Fertility rates

Australia: fertility rates (1921)
Net migration

We need to *estimate* migration data based on difference in population numbers after adjusting for births and deaths.
Net migration

We need to estimate migration data based on difference in population numbers after adjusting for births and deaths.

Demographic growth-balance equation

\[ G_t(x, x + 1) = P_{t+1}(x + 1) - P_t(x) + D_t(x, x + 1) \]
\[ G_t(B, 0) = P_{t+1}(0) - B_t + D_t(B, 0) \]
\[ x = 0, 1, 2, \ldots \]
Net migration

We need to estimate migration data based on difference in population numbers after adjusting for births and deaths.

Demographic growth-balance equation

\[
G_t(x, x + 1) = P_{t+1}(x + 1) - P_t(x) + D_t(x, x + 1)
\]

\[
G_t(B, 0) = P_{t+1}(0) - B_t + D_t(B, 0)
\]

\[x = 0, 1, 2, \ldots\]

Note: “net migration” numbers also include errors associated with all estimates. i.e., a “residual”.
Net migration

Australia: male net migration (1922–2003)
Net migration

Some notation

Let \( y_t(x_i) \) be the observed data (mortality, fertility or net migration) in period \( t \) at age \( x_i \), \( i = 1, \ldots, p, \ t = 1, \ldots, n \).

\[
y_t(x_i) = s_t(x_i) + \sigma_t(x_i) \varepsilon_{t,i}
\]

- \( y_t(x) \) transformed first if necessary
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- $y_t(x)$ transformed first if necessary
- $\varepsilon_{t,i} \overset{iid}{\sim} N(0, 1)$
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- $s_t(x)$ and $\sigma_t(x)$ are smooth functions of $x$. 
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- $\varepsilon_{t,i} \sim_{iid} N(0, 1)$
- $s_t(x)$ and $\sigma_t(x)$ are smooth functions of $x$.
- We need to estimate $s_t(x)$ from the data for $x_1 < x < x_p$. 
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- $y_t(x)$ transformed first if necessary
- $\varepsilon_{t,i} \sim \text{N}(0, 1)$
- $s_t(x)$ and $\sigma_t(x)$ are smooth functions of $x$.
- We need to estimate $s_t(x)$ from the data for $x_1 < x < x_p$.
- We want to forecast \textbf{whole curve} $y_t(x)$ for $t = n + 1, \ldots, n + h$. 
Functional time series model

\[ y_t(x) = s_t(x) + \sigma_t(x)\epsilon_{t,x} \]

\[ s_t(x) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k} \phi_k(x) + e_t(x) \]

where \( \epsilon_{t,x} \sim \text{N}(0, 1) \) and \( e_t(x) \sim \text{N}(0, v(x)) \).
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1. Estimate smooth functions \( s_t(x) \) using nonparametric regression.
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1. Estimate smooth functions \( s_t(x) \) using nonparametric regression.
2. Estimate \( \mu(x) \) as mean \( s_t(x) \) across years.
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Forecasting and the importance of being uncertain

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4. Forecast \( \beta_{t,k} \) using exponential smoothing.
Functional time series model

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4. Forecast \( \beta_{t,k} \) using exponential smoothing.
5. Put it all together to get forecasts of \( y_t(x) \).
Functional time series model

Australia: male death rates (1921–2003)
Functional time series model

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Forecasting and the importance of being uncertain

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Functional time series model

Main effects

Interaction

Beta 1

Beta 2
Forecasting and the importance of being uncertain

Functional time series model

Main effects

Interaction

Mu

Phi 1

Phi 2

Age

0 20 40 60 80 100

−7 −6 −5 −4 −3 −2 −1

0 20 40 60 80 100

0.0 0.1 0.2 0.3 0.4 0.5

Age

0 20 40 60 80 100

−0.3 −0.1 0.1 0.3

Age

0 20 40 60 80 100

−1.5 −0.5 0.0 0.5

Year

1920 1960 2000

Years
**Functional time series model**

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Australia: male death rates (1921–2003)
Functional time series model

Australia: male death rates (1921–2003)
Functional time series model

Australia: male death rate forecasts (2004 and 2023)
Functional time series model

Australia: male death rate forecasts (2004 and 2023)

80% prediction intervals
Forecasting and the importance of being uncertain

**Fertility**

**Main effects**

**Interaction**

**Beta 1**

**Beta 2**

**Phi 1**

**Phi 2**

**Year**

**Age**
Fertility

Australia: forecast fertility rates (2004, 2023)
Forecasting and the importance of being uncertain

Fertility

Australia: forecast fertility rates (2004, 2023)
Migration: male

Migration: male

Main effects

Interaction

Beta 1

Beta 2
Migration: male

Simulation

\[ y_t(x) = s_t(x) + \sigma_t(x)\varepsilon_{t,x} \]

\[ s_t(x) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k} \phi_k(x) + e_t(x) \]

For each of \( m_t^F(x), m_t^M(x), f_t(x), G_t^F(x, x+1), \) and \( G_t^M(x, x+1) \):
Simulation

\[ y_t(x) = s_t(x) + \sigma_t(x)\varepsilon_{t,x} \]
\[ s_t(x) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k} \phi_k(x) + e_{t,x} \]

- For each of \( m_t^F(x) \), \( m_t^M(x) \), \( f_t(x) \), \( G_t^F(x, x+1) \), and \( G_t^M(x, x+1) \):
  - Generate random sample paths of \( \beta_{t,k} \) for \( t = n + 1, \ldots, n + h \) conditional on \( \beta_{1,k}, \ldots, \beta_{n,k} \).
Simulation

\[ y_t(x) = s_t(x) + \sigma_t(x) \varepsilon_{t,x} \]

\[ s_t(x) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k} \phi_k(x) + e_t(x) \]

For each of \( m^F_t(x), m^M_t(x), f_t(x), G^F_t(x, x+1), \) and \( G^M_t(x, x+1) \):

- Generate random sample paths of \( \beta_{t,k} \) for \( t = n + 1, \ldots, n + h \) conditional on \( \beta_{1,k}, \ldots, \beta_{n,k} \).
- Generate random values for \( e_t(x) \) and \( \varepsilon_{t,x} \).
Simulation

\[
y_t(x) = s_t(x) + \sigma_t(x)\varepsilon_{t,x}
\]

\[
s_t(x) = \mu(x) + \sum_{k=1}^{K} \beta_{t,k} \phi_k(x) + e_t(x)
\]

- For each of \( m_t^F(x), m_t^M(x), f_t(x), G_t^F(x, x + 1) \), and \( G_t^M(x, x + 1) \):
  - Generate random sample paths of \( \beta_{t,k} \) for \( t = n + 1, \ldots, n + h \) conditional on \( \beta_{1,k}, \ldots, \beta_{n,k} \).
  - Generate random values for \( e_t(x) \) and \( \varepsilon_{t,x} \).

- Use simulated rates to generate \( B_t(x), D_t^F(x, x + 1), D_t^M(x, x + 1) \) for \( t = n + 1, \ldots, n + h \), assuming deaths and births are Poisson.
Simulation

Demographic growth-balance equation used to get population sample paths.

Demographic growth-balance equation

\[
P_{t+1}(x + 1) = P_t(x) - D_t(x, x + 1) + G_t(x, x + 1)
\]

\[
P_{t+1}(0) = B_t - D_t(B, 0) + G_t(B, 0)
\]

\[x = 0, 1, 2, \ldots\]
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\]

\(x = 0, 1, 2, \ldots\)

- 10000 sample paths of population \(P_t(x)\), deaths \(D_t(x)\) and births \(B_t(x)\) generated for \(t = 2005, \ldots, 2024\) and \(x = 0, 1, 2, \ldots\).
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Demographic growth-balance equation

\[
\begin{align*}
P_{t+1}(x+1) &= P_t(x) - D_t(x, x+1) + G_t(x, x+1) \\
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\end{align*}
\]

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- This allows the computation of the empirical forecast distribution of any demographic quantity that is based on births, deaths and population numbers.
Forecasts of life expectancy at age 0

Forecast female life expectancy

<table>
<thead>
<tr>
<th>Year</th>
<th>Age</th>
<th>Age</th>
<th>Age</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>70</td>
<td>75</td>
<td>80</td>
<td>85</td>
</tr>
<tr>
<td>1980</td>
<td>75</td>
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<tr>
<td>2000</td>
<td>80</td>
<td>85</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>2020</td>
<td>85</td>
<td>90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Forecast male life expectancy

<table>
<thead>
<tr>
<th>Year</th>
<th>Age</th>
<th>Age</th>
<th>Age</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>70</td>
<td>75</td>
<td>80</td>
<td>85</td>
</tr>
<tr>
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<td>80</td>
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<td>90</td>
<td></td>
</tr>
<tr>
<td>2020</td>
<td>85</td>
<td>90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Forecasts of TFR

Forecast Total Fertility Rate

<table>
<thead>
<tr>
<th>Year</th>
<th>TFR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920</td>
<td>1500</td>
</tr>
<tr>
<td>1940</td>
<td>2000</td>
</tr>
<tr>
<td>1960</td>
<td>2500</td>
</tr>
<tr>
<td>1980</td>
<td>3000</td>
</tr>
<tr>
<td>2000</td>
<td>3500</td>
</tr>
<tr>
<td>2020</td>
<td>4000</td>
</tr>
</tbody>
</table>
Forecast population pyramid for 2024, along with 80% prediction intervals. Dashed: actual population pyramid for 2004.
Forecasting and the importance of being uncertain

Population forecasts

Old-age dependency ratio forecasts

<table>
<thead>
<tr>
<th>Year</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920</td>
<td>0.10</td>
</tr>
<tr>
<td>1940</td>
<td>0.15</td>
</tr>
<tr>
<td>1960</td>
<td>0.20</td>
</tr>
<tr>
<td>1980</td>
<td>0.25</td>
</tr>
<tr>
<td>2000</td>
<td>0.30</td>
</tr>
<tr>
<td>2020</td>
<td>0.35</td>
</tr>
</tbody>
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Software and papers:

Outline

1. The dodgy history of forecasting
2. Projections and what-if scenarios
3. Exponential smoothing
4. Forecasting Australia’s population
5. Conclusions
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Slides available from [http://www.robyndman.info/](http://www.robyndman.info/)