MONASH University

MONASH BUSINESS SCHOOL

Forecasting: principles and practice

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3.4 Extras

Outline

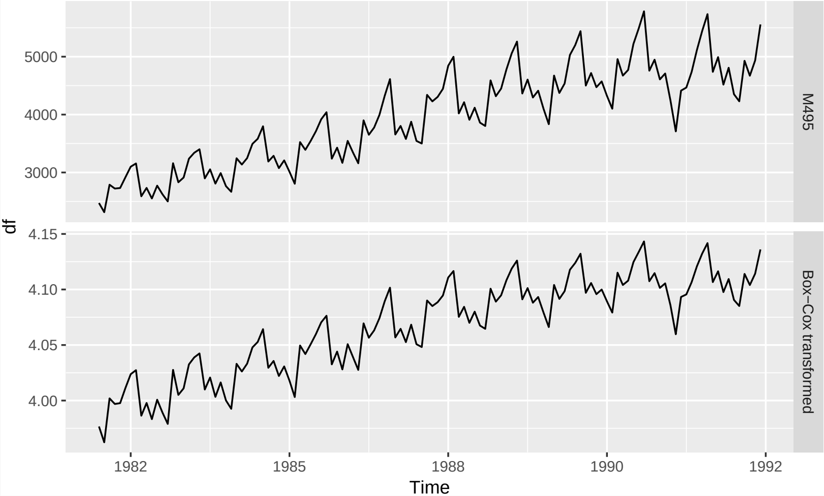
1 Bagged ETS
2 Models for different frequencies
3 Ensuring forecasts stay within limits
4 Forecast combinations
5 Prediction intervals for aggregates
6 Backcasting
7 Missing values
8 Outliers

Bagged ETS

Algorithm: Generating bootstrapped series

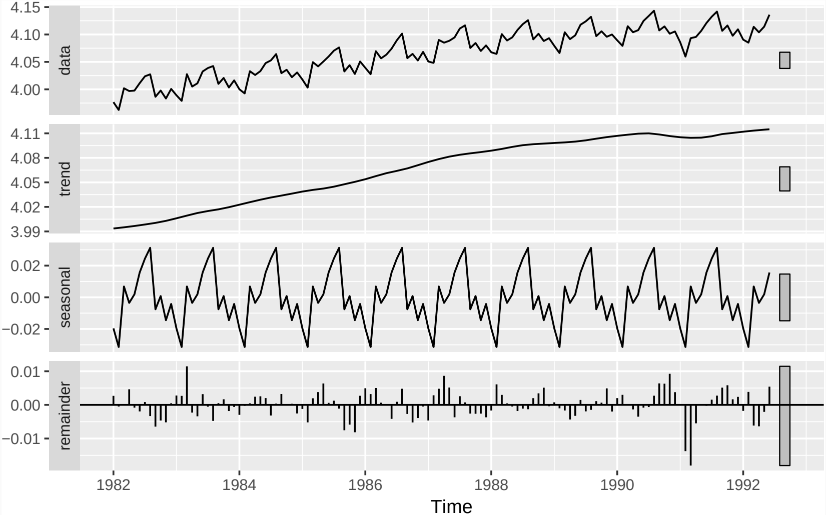
```
bootstrap ← function(ts, num.boot) {
  lambda ← BoxCox.lambda(ts, min=0, max=1)
  ts.bc ← BoxCox(ts, lambda)
  if(ts is seasonal) {
    [trend, seasonal, remainder] ← stl(ts.bc)
  }
  else {
    seasonal ← 0
    [trend, remainder] ← loess(ts.bc)
  }
  recon.series[1] ← ts
  for(i in 2:num.boot) {
    boot.sample[i] ← MBB(remainder)
    recon.series.bc[i] ← trend + seasonal + boot.sample[i]
    recon.series[i] ← InvBoxCox(recon.series.bc[i], lambda)
  }
  return(recon.series)
```

Bagged ETS



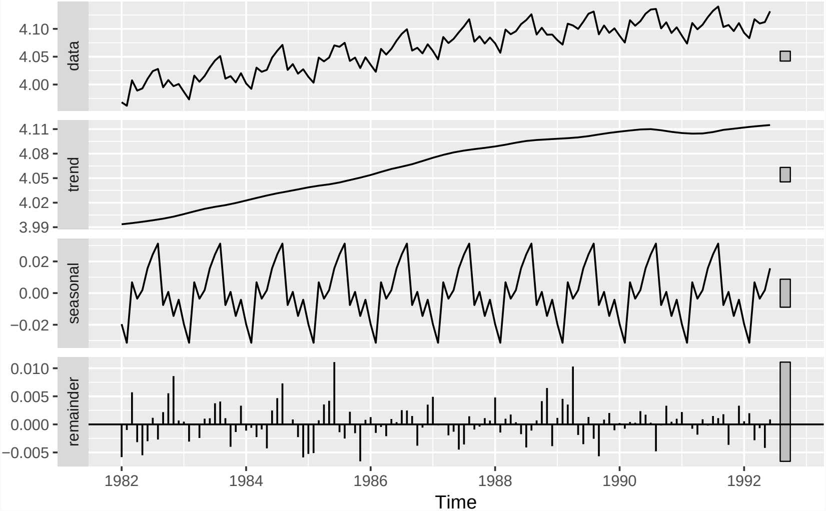
M495
Box–Cox transformed
df
5000
4000
3000
4.15
4.10
4.05
4.00
1982
1985
1988
1990
1992
Time

Bagged ETS



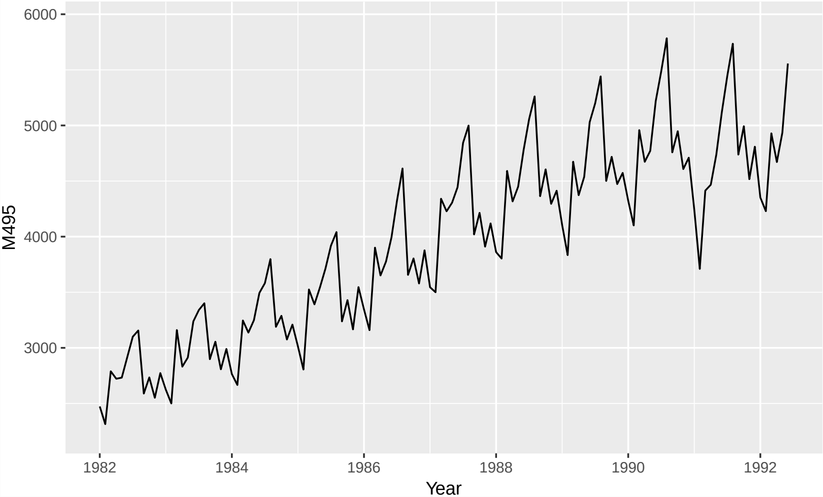
4.15
4.10
4.05
4.00
data
4.11
4.08
4.05
4.02
3.99
trend
0.02
0.00
−0.02
seasonal
0.01
0.00
−0.01
remainder
1982
1984
1986
1988
1990
1992
Time

Bagged ETS

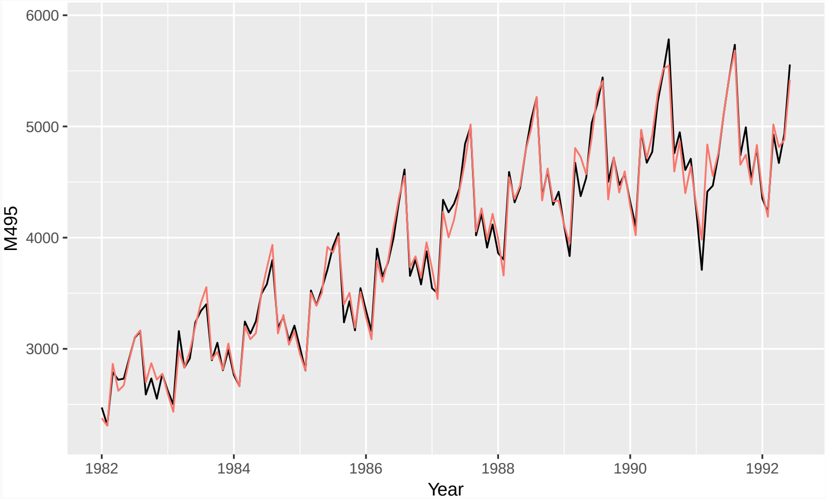


4.10
4.05
4.00
data
4.11
4.08
4.05
4.02
3.99
trend
0.02
0.00
−0.02
seasonal
0.010
0.005
0.000
−0.005
remainder
1982
1984
1986
1988
1990
1992
Time

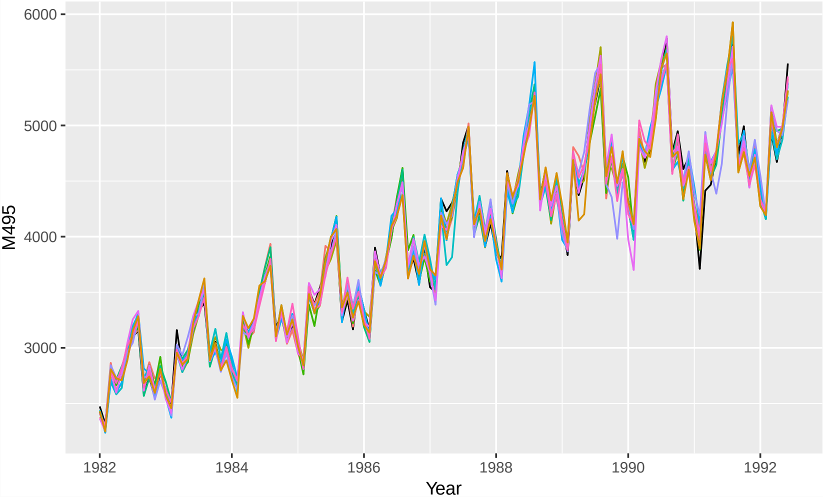
Bagged ETS



Bagged ETS

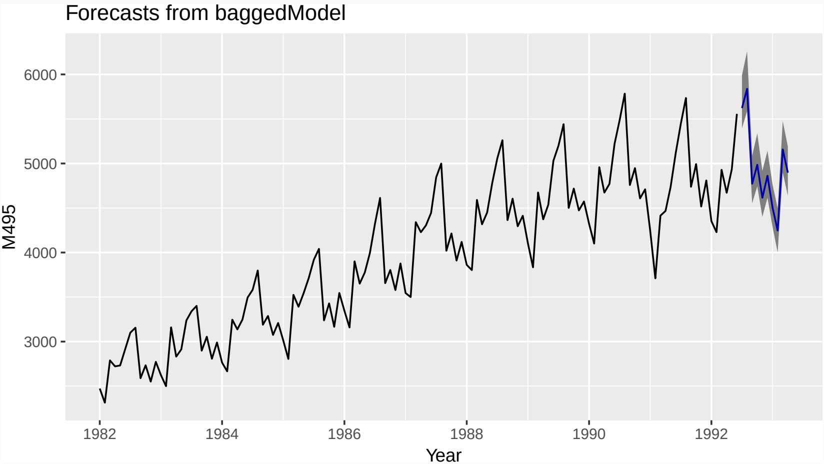


Bagged ETS

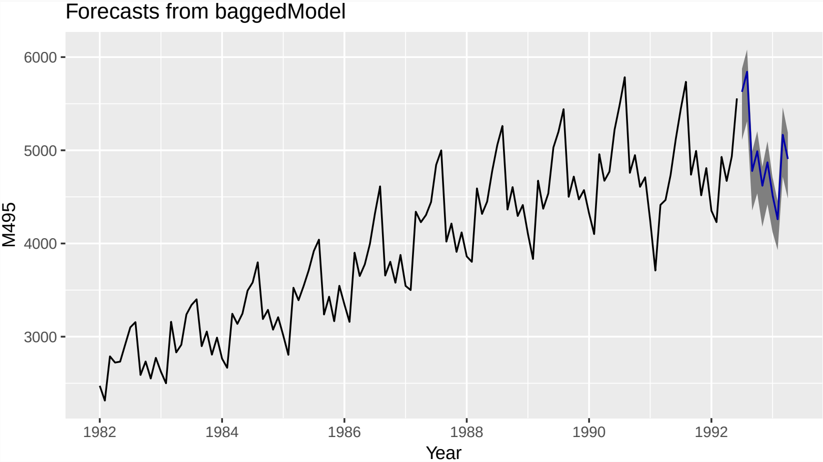


Bagged ETS

```r
baggedETS(Mcomp::M3[[1896]]$x) %>%
  forecast %>% autoplot +
  xlab("Year") + ylab("M495")
```



Bagged ETS



- Intervals show range of point forecasts
- They are not prediction intervals

Outline

Models for different frequencies

Models for annual data

- ETS, ARIMA, Dynamic regression

Models for different frequencies

Models for annual data

- ETS, ARIMA, Dynamic regression

Models for quarterly data

- ETS, ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA

Models for different frequencies

Models for annual data

- ETS, ARIMA, Dynamic regression

Models for quarterly data

- ETS, ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA

Models for monthly data

- ETS, ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA

Models for different frequencies

Models for weekly data

- ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA, TBATS

Models for different frequencies

Models for weekly data

- ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA, TBATS

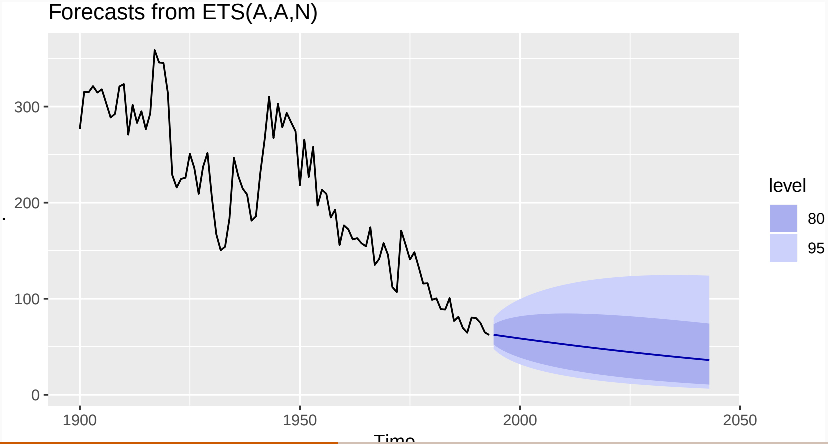
Models for daily, hourly and other sub-daily data

- ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA, TBATS

Outline

Positive forecasts

```r
eggs %>%
  ets(model="AAN", damped=FALSE, lambda=0) %>%
  forecast(h=50, biasadj=TRUE) %>%
  autoplot()
```



Forecasts constrained to an interval

Suppose egg prices constrained to lie within $a = 50$ and $b = 400$.

Transform data using scaled logit transform:

$$y = \log\left(\frac{x - a}{b - x}\right),$$

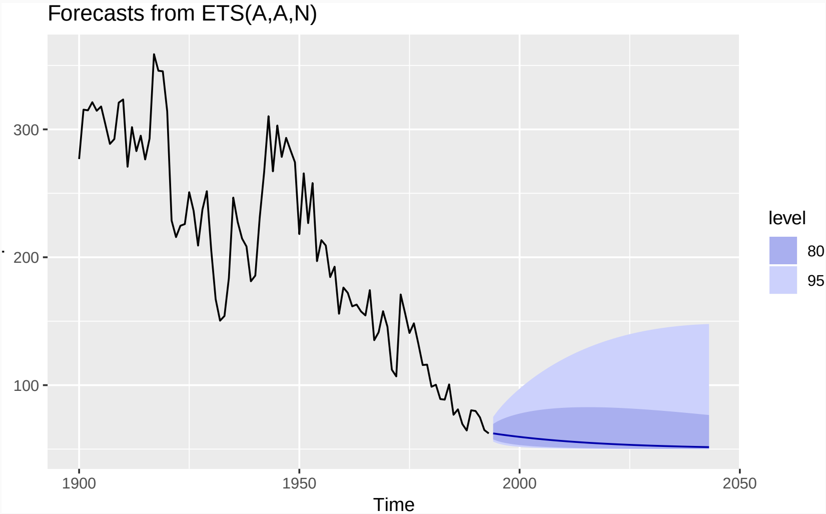
where x is on the original scale and y is the transformed data. To reverse the transformation, we will use

$$x = \frac{(b - a)e^y}{1 + e^y} + a.$$

Forecasts constrained to an interval

```r
# Bounds
a <- 50
b <- 400
# Transform data and fit model
fit <- log((eggs-a)/(b-eggs)) %>%
  ets(model="AAN", damped=FALSE)
fc <- forecast(fit, h=50)
# Back-transform forecasts
fc[["mean"]] <- (b-a)*exp(fc[["mean"]]) /
  (1+exp(fc[["mean"]])) + a
fc[["lower"]] <- (b-a)*exp(fc[["lower"]]) /
 (1+exp(fc[["lower"]])) + a
fc[["upper"]] <- (b-a)*exp(fc[["upper"]]) /
 (1+exp(fc[["upper"]])) + a
fc[["x"]] <- eggs
autoplot(fc)
```

Forecasts constrained to an interval



Outline

Forecast combinations

Clemen (1989)

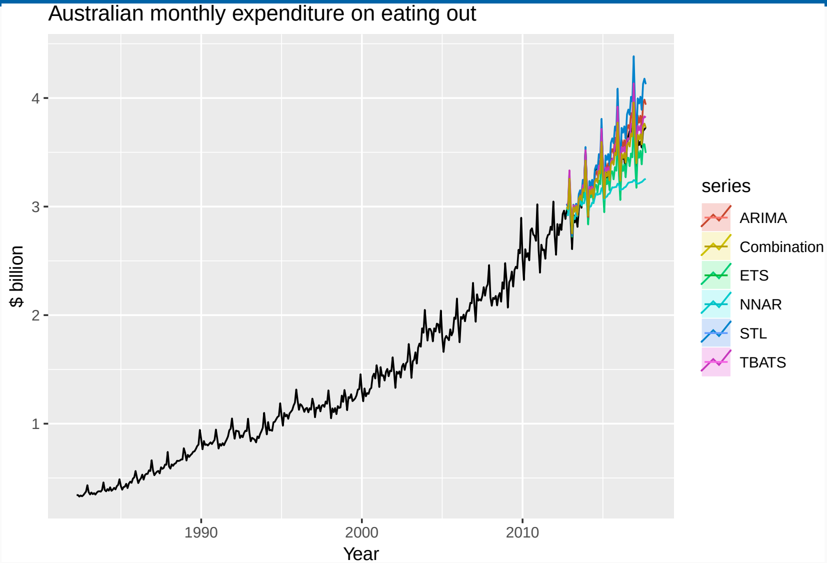
"The results have been virtually unanimous: combining multiple forecasts leads to increased forecast accuracy. …In many cases one can make dramatic performance improvements by simply averaging the forecasts."

Forecast combinations

```
train <- window(auscafe, end=c(2012,9))
h <- length(auscafe) - length(train)
ETS <- forecast(ets(train), h=h)
ARIMA <- forecast(auto.arima(train, lambda=0, biasadj=TRUE),
  h=h)
STL <- stlf(train, lambda=0, h=h, biasadj=TRUE)
NNAR <- forecast(nnetar(train), h=h)
TBATS <- forecast(tbats(train, biasadj=TRUE), h=h)
Combination <- (ETS[["mean"]] + ARIMA[["mean"]] +
  STL[["mean"]] + NNAR[["mean"]] + TBATS[["mean"]])/5

autoplot(auscafe) +
  autolayer(ETS, series="ETS", PI=FALSE) +
  autolayer(ARIMA, series="ARIMA", PI=FALSE) +
  autolayer(STL, series="STL", PI=FALSE) +
  autolayer(NNAR, series="NNAR", PI=FALSE) +
  autolayer(TBATS, series="TBATS", PI=FALSE) +
  autolayer(Combination, series="Combination") +
  xlab("Year") + ylab("$ billion") +
  ggtitle("Australian monthly expenditure on eating out")
```

Forecast combinations



Forecast combinations

```r
c(ETS = accuracy(ETS, auscafe)["Test set","RMSE"],
  ARIMA = accuracy(ARIMA, auscafe)["Test set","RMSE"],
  STL-ETS = accuracy(STL, auscafe)["Test set","RMSE"],
  NNAR = accuracy(NNAR, auscafe)["Test set","RMSE"],
  TBATS = accuracy(TBATS, auscafe)["Test set","RMSE"],
  Combination =
    accuracy(Combination, auscafe)["Test set","RMSE"])
```

```
##         ETS       ARIMA     STL-ETS        NNAR
##     0.13700     0.12146     0.21446     0.29801
##       TBATS Combination
##     0.09406     0.07090
```

Outline

Prediction intervals for aggregates

```r
# First fit a model to the data
fit <- ets(gas/1000)
# Forecast six months ahead
fc <- forecast(fit, h=6)
sum(fc[["mean"]][1:6])
```

```
## [1] 281.8
```

```r
# Simulate 10000 future sample paths
nsim <- 10000
h <- 6
sim <- numeric(nsim)
for(i in seq_len(nsim))
  sim[i] <- sum(simulate(fit, future=TRUE, nsim=h))
mean(sim)
```

```
## [1] 281.9
```

Prediction intervals for aggregates

```r
#80% interval:
quantile(sim, prob=c(0.1, 0.9))
```

```
##   10%   90%
## 262.9 300.9
```

```r
#95% interval:
quantile(sim, prob=c(0.025, 0.975))
```

```
##  2.5% 97.5%
## 253.2 311.2
```

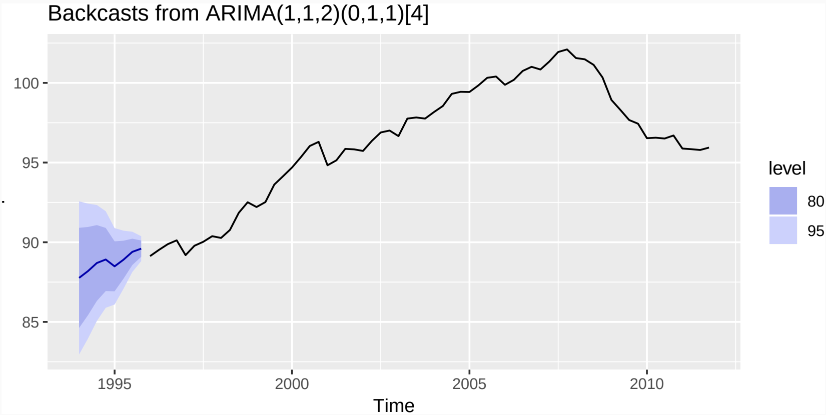
Outline

Backcasting

```r
# Function to reverse time
reverse_ts <- function(y)
{
  ts(rev(y), start=tsp(y)[1L], frequency=frequency(y))
}
# Function to reverse a forecast
reverse_forecast <- function(object)
{
  h <- length(object[["mean"]])
  f <- frequency(object[["mean"]])
  object[["x"]] <- reverse_ts(object[["x"]])
  object[["mean"]] <- ts(rev(object[["mean"]]),
    end=tsp(object[["x"]])[1L]-1/f, frequency=f)
  object[["lower"]] <- object[["lower"]][h:1L,]
  object[["upper"]] <- object[["upper"]][h:1L,]
  return(object)
}
```

Backcasting

```
euretail %>% reverse_ts() %>%
  auto.arima() %>%
  forecast() %>% reverse_forecast() -> bc
autoplot(bc) +
  ggtitle(paste("Backcasts from",bc[["method"]]))
```



Outline

Missing values

Functions which can handle missing values

- `auto.arima()`, `Arima()`
- `tslm()`
- `nnetar()`

Models which cannot handle missing values

- `ets()`
- `stl()`
- `stlf()`
- `tbats()`

Missing values

Functions which can handle missing values

- `auto.arima()`, `Arima()`
- `tslm()`
- `nnetar()`

Models which cannot handle missing values

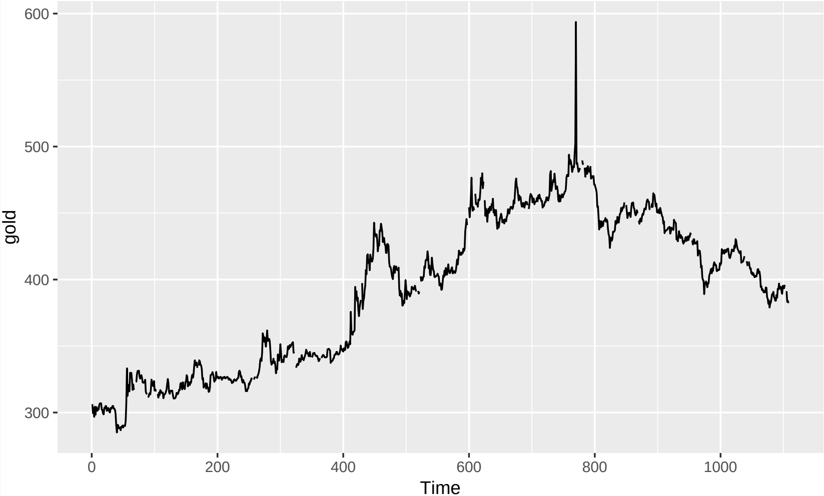
- `ets()`
- `stl()`
- `stlf()`
- `tbats()`

What to do?

1. Model section of data after last missing value.
2. Estimate missing values with `na.interp()`.

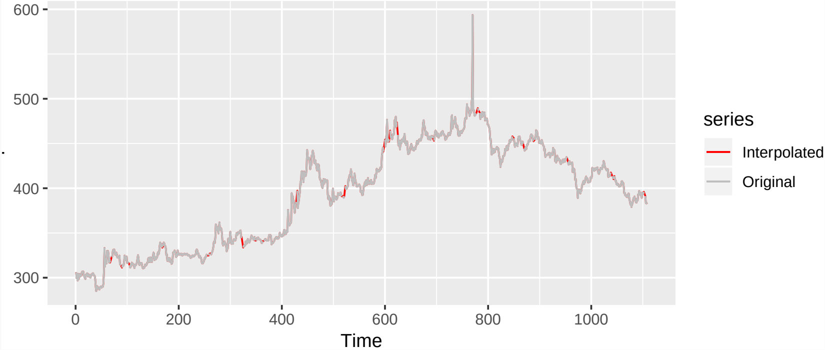
Missing values

```
autoplot(gold)
```



Missing values

```
gold %>% na.interp() %>%
  autoplot(series="Interpolated") +
    autolayer(gold, series="Original") +
    scale_color_manual(
      values=c(Interpolated="red",Original="gray"))
```

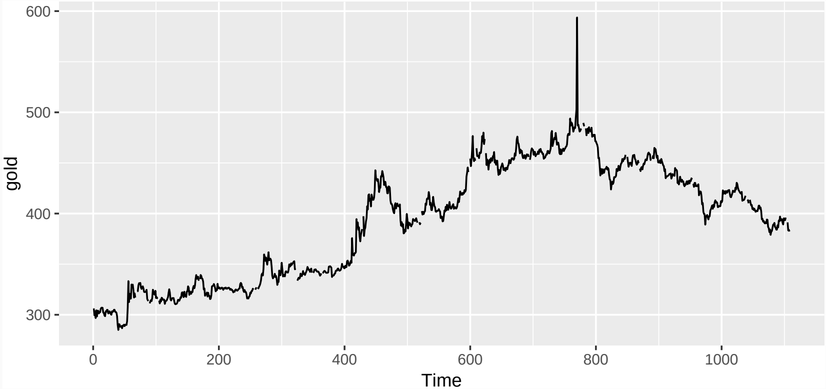


Outline

1. Bagged ETS
2. Models for different frequencies
3. Ensuring forecasts stay within limits
4. Forecast combinations
5. Prediction intervals for aggregates
6. Backcasting
7. Missing values
8. **Outliers**

Outliers

```
autoplot(gold)
```



Outliers

```r
tsoutliers(gold)
```

```
## $index
## [1] 770
##
## $replacements
## [1] 494.9
```

Outliers

```
gold %>% tsclean() %>% autoplot()
```